EXPLORATION

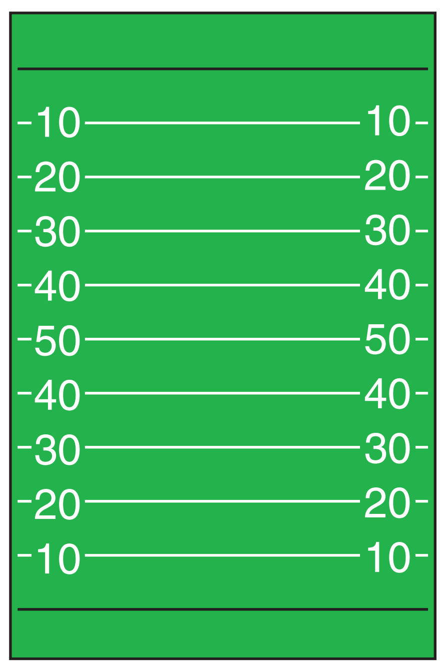
1-1 Sets of Numbers

A set is a group of items, such as a group of numbers. For example, the numbers that appear on a telephone keypad form a set. The items in a set are called elements.

1. One method of describing a set is to list its elements inside a pair of braces, { }. Use this notation to write the set of numbers found on a telephone keypad.
2. You can also describe a set by describing its properties. Describe the set of numbers found on a telephone keypad without listing them.
3. List four numbers that are NOT included in the set you described in Problem 2.

THINK AND DISCUSS

4. **Explain** how terms such as natural numbers, whole numbers, and integers can help you describe a set.
5. **Describe** the set of numbers that appear on a football field.



EXPLORATION

1-2 Properties of Real Numbers

You may use a calculator to help you with the following problems.

Tell whether each statement is true or false.

1. $6 + (-2) = (-2) + 6$
2. $8 - (-2) = (-2) - 8$
3. $5 - 4 - 1 = 1 - 4 - 5$
4. $20 \div 5 \div 2 = 2 \div 5 \div 20$
5. $3(7) = 7(3)$
6. $7 + 3 + 2 = 2 + 7 + 3$
7. $9 \div 3 = 3 \div 9$
8. $10 \cdot 20 \cdot 30 = 30 \cdot 20 \cdot 10$
9. Based on your answers to Problems 1–8, for which operations (addition, subtraction, multiplication, and division) does the order of the numbers make a difference in the result?

Tell whether each statement is true or false.

10. $(8 \div 4) \div 2 = 8 \div (4 \div 2)$
11. $(9 + 4) + 3 = 9 + (4 + 3)$
12. $(5 - 4) - 1 = 5 - (4 - 1)$
13. $21 \cdot (32 \cdot 43) = (21 \cdot 32) \cdot 43$
14. Based on your answers to Problems 10–13, for which operations (addition, subtraction, multiplication, and division) does the grouping of the numbers make a difference in the result?

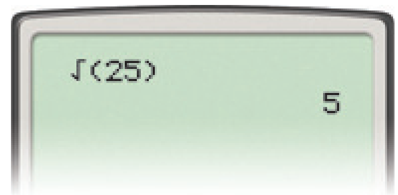
THINK AND DISCUSS

15. **Discuss** how the properties you explored above might help you find $13 \cdot 20 \cdot 5$ by using mental math.
16. **Discuss** whether it would matter which operation you did first if you had more than one operation in a problem.

EXPLORATION

1-3 Square Roots

You can use a calculator to help you investigate some properties of square roots.



To enter square roots, press 2nd x^2 ($\sqrt{\ }$).

1. Find the value of each expression in the table. Round to the nearest thousandth.

Expression	Value	Expression	Value
$\sqrt{3} \cdot \sqrt{5}$		$\sqrt{15}$	
$\sqrt{7} \cdot \sqrt{2}$		$\sqrt{14}$	
$\sqrt{50} \cdot \sqrt{2}$		$\sqrt{100}$	

2. Based on your answers to Problem 1, what can you say about $\sqrt{a} \cdot \sqrt{b}$?

3. Find the value of each expression in the table. Round to the nearest thousandth.

Expression	Value	Expression	Value
$\frac{\sqrt{15}}{\sqrt{5}}$		$\sqrt{3}$	
$\frac{\sqrt{60}}{\sqrt{12}}$		$\sqrt{5}$	
$\frac{\sqrt{100}}{\sqrt{4}}$		$\sqrt{25}$	

4. Based on your answers to Problem 3, what can you say about $\frac{\sqrt{a}}{\sqrt{b}}$?

THINK AND DISCUSS

5. **Explain** how you can use one of the properties you discovered above to simplify $\sqrt{5} \cdot \sqrt{20}$ by using mental math.

EXPLORATION

1-4 Simplifying Algebraic Expressions

Whenever you simplify numerical or algebraic expressions, it is important to perform the operations in the correct order.

Simplify each expression. In each case, think about the order in which you should do the operations.

1. $(7 + 2)^2 - 5$
2. $[(2 - 10) + (4 - 1)]^2$
3. $3 - 5(21 - 19)^3$
4. $[(3^3 - 1) \div 13]^2$
5. Check your answers to Problems 1–4 by entering each expression into a graphing calculator.
6. The order of operations tells you the sequence in which you should perform operations. Fill in the flow chart by arranging the steps given below in the correct order.
   - *Add and subtract from left to right.*
   - *Parentheses*
   - *Multiply and divide from left to right.*
   - *Exponents*

Order of Operations

1.
2.
3.
4.

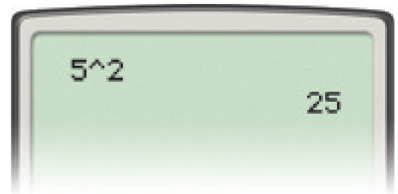
THINK AND DISCUSS

7. **Explain** how you can use the order of operations to evaluate $x^2 - (x + y)$ when $x = -3$ and $y = 4$.
8. **Discuss** whether $x - (y + 1)$ and $x - y + 1$ are equivalent expressions. How does the order of operations determine the value of the expressions?

EXPLORATION

1-5 Properties of Exponents

You can use a calculator to help you investigate some properties of exponents. Use the ^ key to indicate an exponent.



1. Find the value of each expression in the table.

Expression	Value	Expression	Value
$2^2 \cdot 2^3$		2^5	
$4^3 \cdot 4^3$		4^6	
$10^2 \cdot 10^5$		10^7	

2. Based on your answers to Problem 1, what can you say about $a^m \cdot a^n$?

3. Find the value of each expression in the table.

Expression	Value	Expression	Value
$\frac{2^5}{2^2}$		2^3	
$\frac{5^3}{5^1}$		5^2	
$\frac{10^6}{10^3}$		10^3	

4. Based on your answers to Problem 3, what can you say about $\frac{a^m}{a^n}$?

THINK AND DISCUSS

5. Explain how you can use one of the properties you discovered above to simplify $\frac{12^7}{12^6}$ by using mental math.

EXPLORATION

1-6 Relations and Functions

A *relation* is a pairing of values that can be written as a set of ordered pairs. A *function* is a special type of relation. Use the information below to explore what makes a function different from other types of relations.

Functions

x	*y*
0	1
1	1
2	1
3	1

x	*y*
0	2
2	1
5	−1
9	−4

x	*y*
−2	3
0	2
2	4
4	5

Not Functions

x	*y*
0	1
0	2
1	3
1	4

x	*y*
1	−5
2	−4
3	−5
1	−4

x	*y*
−2	1
2	4
2	−4
−2	−1

1. Examine the *x*-values in the tables. Make a conjecture about the *x*-values in a function.

2. Does your conjecture apply to *y*-values as well? Explain.

3. Use your conjecture to determine whether the ordered pairs in each table represent a function.

a.

x	−2	−1	0	−1	−2
y	5	4	3	2	1

b.

x	−2	−1	0	1	2
y	3	5	7	5	3

THINK AND DISCUSS

4. **Describe** what makes a relation a function.

EXPLORATION

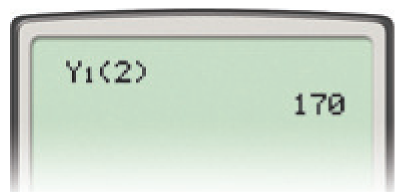
1-7 Function Notation

Felicia is driving to see her parents. When she starts, she is at mile marker 50 on the interstate. She drives at an average speed of 60 mi/h. In the table, t stands for time in hours, and $d(t)$ stands for the mile marker number at time t.

t	$d(t)$
0	$d(0) = 50$
0.5	
1	$d(1) = 110$
1.5	
2	

1. Complete the table to show the mile marker where Felicia is at each half hour.

You can also use a calculator to determine Felicia's location at various times. Enter the function rule by pressing [Y=] and entering **50 + 60X** as shown. Then press [2nd] [MODE] (QUIT) to return to the home screen. Select the function you entered by pressing [VARS], scrolling right to **Y-VARS,** selecting **1:Function,** and choosing **1:Y1.** You can evaluate the function for any x-value by entering a value in parentheses, as shown.



2. Use your calculator to find **Y1(3.5).**
3. Explain what **Y1(3.5)** represents.
4. Felicia's exit is at mile marker 362. Determine how many hours it will take her to get there.

THINK AND DISCUSS

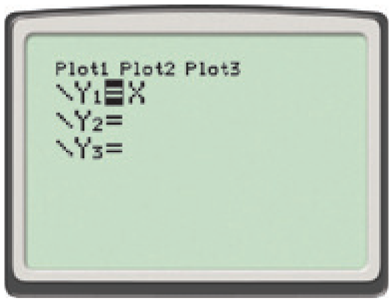
5. **Describe** the method you used to solve Problem 4.
6. **Discuss** whether **Y1(–3)** has meaning in the context of this problem.

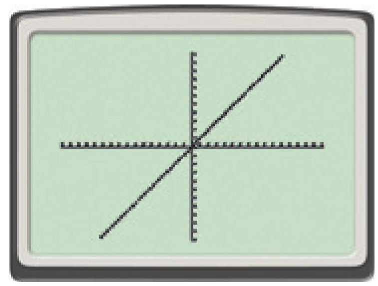
EXPLORATION

1-8 Exploring Transformations

You can explore transformations by using the graph of a line on a graphing calculator.

Graph $y = x$ on a graphing calculator by pressing Y= and entering **Y1 = X** as shown. Then press GRAPH to see the graph.





1. Enter and graph **Y2 = X + 5**. Describe the graph of **Y2** as compared to the graph of **Y1**.

2. Enter and graph **Y3 = X − 5**. Describe the graph of **Y3** as compared to the graph of **Y1**.

3. Make a conjecture about how a change in the value of k in the equation $y = x + k$ affects the equation's graph.

THINK AND DISCUSS

4. **Explain** what equation you would use to move the graph of $y = x$ down 3 units.

5. **Describe** how the graph of $y = x + 100$ differs from the graph of $y = x$.

EXPLORATION

1-9 Introduction to Parent Functions

Although there are an infinite number of functions, many functions can be grouped into families that have similar characteristics.

1. Use your calculator to graph $y = x^2$ by pressing Y=, entering **Y1 = X²**, and pressing GRAPH. Describe the graph.

2. Enter and graph **Y2 = X² + 2**. Describe the graph of **Y2** as compared to the graph of **Y1**.

3. Enter and graph **Y3 = 2X²**. Compare this graph to the graphs from Problems 1 and 2.

4. Enter and graph **Y4 = 2X² − 6**. Compare this graph to the graphs above.

5. Make a conjecture about the shape of the graphs of functions of the form $y = ax^2 + b$.

THINK AND DISCUSS

6. **Explain** what the graphs of functions of the form $y = mx + b$ look like.

EXPLORATION

2-1 Solving Linear Equations and Inequalities

A group of ten friends are on vacation in Canada. They decide to rent an ice-skating rink so they can play hockey.

Rink Rental
$50 plus $25 per hour

Equipment (per person)
$5 + $1.50 per hour

1. Everyone in the group needs to rent equipment. Write an expression for the total cost of the equipment for x hours.
2. Write an expression for the total cost of the rink and the equipment for x hours.
3. Use your expression to help you complete the table.

Time (h)	1	2	3	4	5	6	7	8
Total Cost ($)								

4. The rink and equipment can be rented only in whole-hour increments. The friends want to spend less than $350. For how many hours can they rent the rink and equipment?

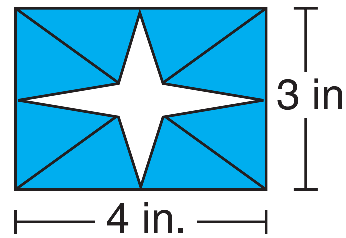
THINK AND DISCUSS

5. **Describe** another method you could use to solve Problem 4, other than making a table.
6. **Explain** how the solution to Problem 4 would change if a 10% tax were added to the bill.

EXPLORATION

2-2 Proportional Reasoning

Sara is a graphic artist. She has designed this rectangular logo for a chain of supermarkets.



1. The logo will be available in different sizes, as shown in the table. Describe any patterns formed by the numbers in the table.

Length (in.)	4	5	6	7	8
Height (in.)	3	3.75	4.5	5.25	6

2. What should the height of the logo be if the length is 9 inches?

3. What should the length of the logo be if the height is 15 inches?

4. Sara sees the logo on a brochure. She wants to check that it is the correct shape, so she measures the length and height. How can she use the measurements to check the shape?

THINK AND DISCUSS

5. **Discuss** how you solved Problems 2 and 3. Describe as many different solution methods as you can.

6. **Give** an equation that relates the length and height of the logo.

EXPLORATION

2-3 Graphing Linear Functions

Jorge is filling the stock tank at his ranch. He starts when the water in the tank is 5 feet deep. The water rises 1.5 inches per minute.

1. Write an equation that gives the depth of the water in inches y after x minutes.

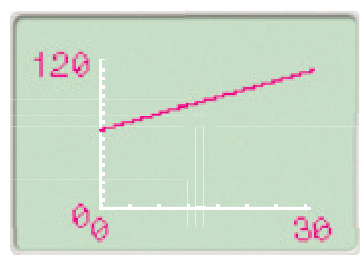
2. Complete the table.

Time (min)	0	6	12	18	24	30
Water Depth (in.)	60					

3. Find the successive differences in the water depths, as indicated here. What do you notice?

Time (min)	0	6	12	18	24	30
Water Depth (in.)	60					

4. Use your graphing calculator to graph the equation. What is the shape of the graph?



THINK AND DISCUSS

5. **Describe** several different methods you could use to find the depth of the water after 36 minutes.

6. **Explain** how you can find out how long it will take to fill the tank if it can hold 10 feet of water.

EXPLORATION

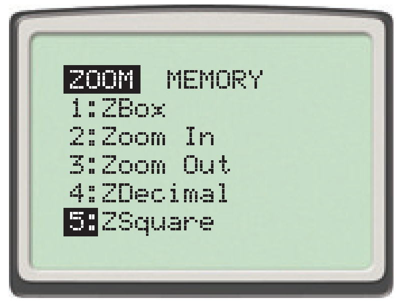
2-4 Writing Linear Functions

Use a graphing calculator for this Exploration.

1. Press Y= and enter the following functions as **Y1** and **Y2**.

   $y = \frac{3}{4}x - 2$ (Enter **3/4X−2**).

   $y = -\frac{4}{3}x + 1$ (Enter **−4/3X+1**).



2. Press ZOOM and choose **ZStandard**. Then press ZOOM and **ZSquare** to graph the functions in a square window.

3. What appears to be true about the graphs?

4. Examine the two equations. What do you notice about their slopes?

5. Repeat Problems 1–4 but instead use the two linear equations below.

   $y = -4x + 3$

   $y = \frac{1}{4}x - 1$

THINK AND DISCUSS

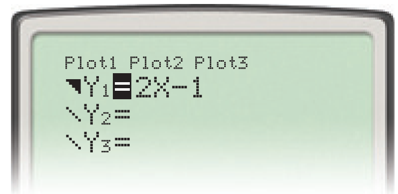
6. **Make** a generalization based on your findings.

7. **Explain** why you needed to set the shape of the viewing window using **ZSquare** for this Exploration.

EXPLORATION

2-5 Linear Inequalities in Two Variables

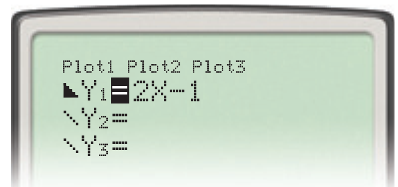
You can use a graphing calculator to explore inequalities.



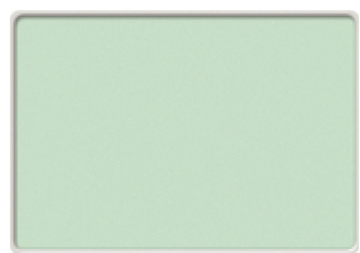
1. Press Y= and enter the function $y = 2x - 1$ as **Y1**.

2. You can graph the inequality $y \geq 2x - 1$ in the following way: Use the arrow keys to move the cursor to the extreme left of the line on which you entered **Y1**.
   Press ENTER until you see the shaded triangle shown on the graph at right.
   Press GRAPH .

3. What points along the y-axis are in the shaded region?



4. What points along the vertical line $x = 2.5$ are in the shaded region?



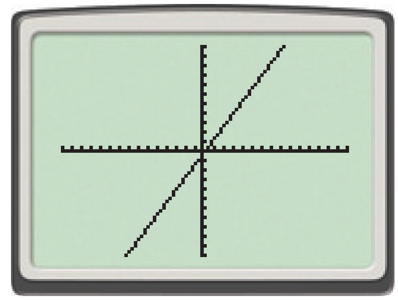
5. You can enter $y \leq 2x - 1$ by moving the cursor to the extreme left of the line on which you entered the function.
   Press ENTER until you see the shaded triangle shown on the graph at right.
   Press GRAPH.

THINK AND DISCUSS

6. **Describe** what would happen if you graphed $y \geq 2x - 1$ and $y \leq 2x - 1$ on the same coordinate plane.
7. **Explain** how you can know whether the origin will be in the shaded region when you graph $y \geq 2x - 1$.

EXPLORATION

2-6 Transforming Linear Functions



Use a graphing calculator to explore transformations of linear functions. Begin by graphing $y = 2x$.

1. Graph $y = 2x + 3$ on the same coordinate plane as $y = 2x$. How does the graph of this function compare to that of $y = 2x$?
2. Graph $y = 2x - 2$ on the same coordinate plane as $y = 2x$. How does the graph of this function compare to that of $y = 2x$?
3. Graph $y = 2(x - 3)$ on the same coordinate plane as $y = 2x$. How does the graph of this function compare to that of $y = 2x$?
4. Graph $y = 2(x + 2)$ on the same coordinate plane as $y = 2x$. How does the graph of this function compare to that of $y = 2x$?

THINK AND DISCUSS

5. **Describe** how the graph of $y = -4x + k$ is related to the graph of $y = -4x$.
6. **Explain** how you can use what you discovered to quickly graph $y = 3(x - 5)$.

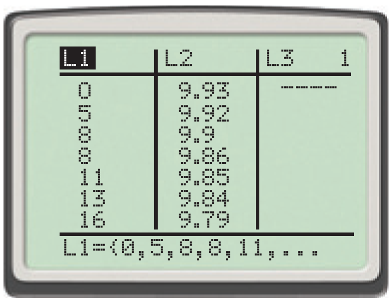
EXPLORATION

2-7 Curve Fitting with Linear Models

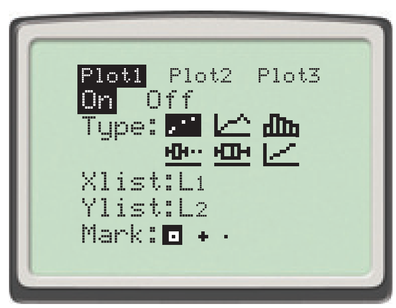
The table shows the dates and winning times for record holders in the men's 100-meter sprint. You can use a graphing calculator to help you see trends in the data.

Date	Record Holder	Time (s)
1983	Calvin Smith	9.93
1988	Carl Lewis	9.92
1991	Leroy Burrell	9.90
1991	Carl Lewis	9.86
1994	Leroy Burrell	9.85
1996	Donovan Bailey	9.84
1999	Maurice Green	9.79
2002	Tim Montgomery	9.78
2005	Asafa Powell	9.77

1. Enter the years in list **L1** by pressing STAT and then 1. Let 1983 be year 0 and 2005 be year 22. Then enter the times in list **L2**.



2. Make a scatter plot in the following way: Press 2nd Y=. Then select **Plot1** and set up the plot as shown. When you are done, press GRAPH. Adjust the viewing window as needed.



THINK AND DISCUSS

3. **Describe** the pattern of the data in the scatter plot.
4. **Explain** how you could use the scatter plot to make predictions about future records in the 100-meter sprint.

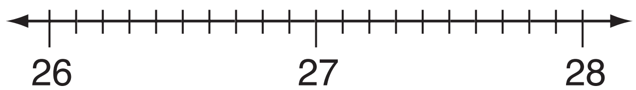
EXPLORATION

2-8 Solving Absolute-Value Equations and Inequalities

A carpenter prepares several wooden dowels whose lengths are 27 cm $\pm$ 0.3 cm.

1. What is the range of possible lengths for the dowels?

2. Use the number line to show the range of possible lengths.



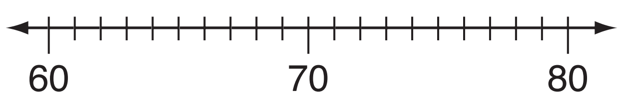
3. The carpenter writes the following to show the range of possible lengths.

$-0.3 \leq x - 27 \leq 0.3$

This is equivalent to the two inequalities $-0.3 \leq x - 27$ and $x - 27 \leq 0.3$. Solve the inequalities to show that they represent the same range of lengths.

THINK AND DISCUSS

4. Describe different ways to write the range of values shown on this number line.

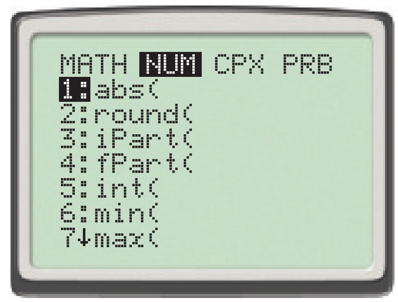


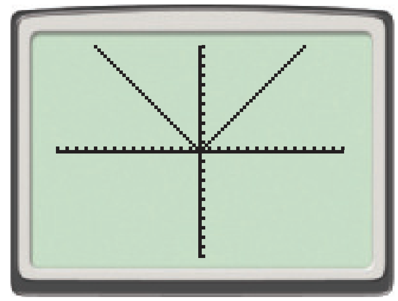
5. Demonstrate how to write an inequality like the one in Problem 3 for the following situation: The weight of a Great Dane is within 5 pounds of 111 pounds.

EXPLORATION

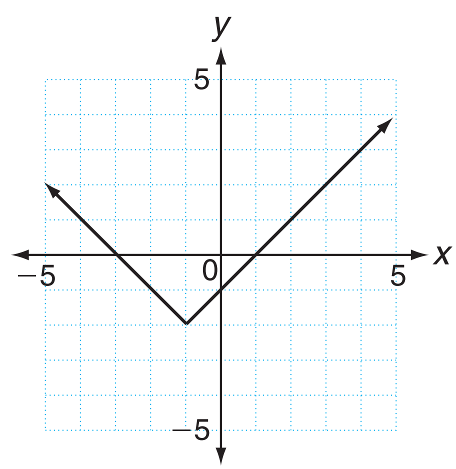
2-9 Absolute-Value Functions

You can use a graphing calculator to explore transformations of the function $y = |x|$.





1. Press **Y=**. To enter the function $y = |x|$ press **MATH** and then select **NUM** from the menu at the top of the screen. Then press **ENTER**. You can then complete **Y1** as **Y1=abs(X)**.
2. Graph the function in a square window.
3. Graph $y = |x| + 3$ on the same coordinate plane as $y = |x|$. How does the graph of $y = |x| + 3$ compare to that of $y = |x|$?
4. Graph $y = |x - 4|$. Describe the graph.
5. Write a function for the graph shown here. Check your answer by graphing the function on your graphing calculator.



THINK AND DISCUSS

6. **Describe** how you would sketch a graph of $y = |x - 1| + 5$.
7. **Explain** how you know whether a graph of an absolute value function that horizontally shifts from $y = |x|$ shifts to the left or to the right.

EXPLORATION

3-1 Using Graphs and Tables to Solve Linear Systems

Felipe is driving 55 mi/h on the interstate and is currently at mile marker 125. Gina is driving in the same direction at 65 mi/h and is currently at mile marker 85.

1. Complete the table.

Time (h)	Felipe's Location	Gina's Location
0	125	85
1		
2		
3		
4		
5		

2. Write an equation for Felipe's location. Let y represent the mile marker number, and let x represent the number of hours.

3. Write an equation for Gina's location. Let y represent the mile marker number, and let x represent the number of hours.

4. How long does it take for Gina to catch Felipe? At what location does she catch him?

THINK AND DISCUSS

5. **Explain** how you could write a system of equations to represent this problem.

6. **Describe** how you could use a graph to solve this problem.

EXPLORATION

3-2 Using Algebraic Methods to Solve Linear Systems

The monthly cost *y* for a factory to manufacture stereo headphones is $12,600 plus $17.50 per headphone. The headphones are then sold for $25 each.

1. Write an equation for the cost of manufacturing headphones. Let *y* represent the total cost, and let *x* represent the number of headphones manufactured.

2. Write an equation for the revenue from the headphones. Let *y* represent the revenue and let *x* represent the number of headphones manufactured.

3. Set the expressions for cost and revenue equal to each other and then solve the equation for *x*.

4. Interpret your solution. What is the result when the factory manufactures this number of headphones?

THINK AND DISCUSS

5. **Determine** the number of headphones the factory should manufacture to make a profit.

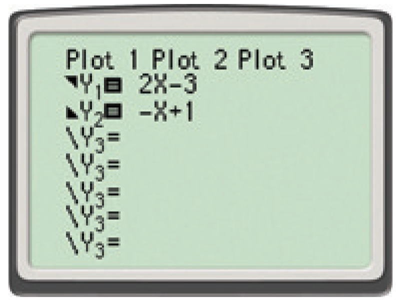
6. **Discuss** the advantages of solving this problem algebraically rather than by using a table or graph.

EXPLORATION

3-3 Solving Systems of Linear Inequalities

You can use a graphing calculator to graph a system of inequalities.

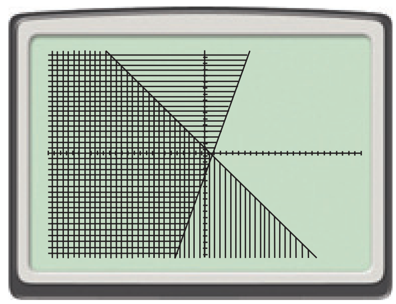
The Y= menu shows the inequalitites $y \geq 2x - 3$ and $y \leq -x + 1$. Note that the symbol to the left of **Y** indicate shading.



1. Graph only $y \geq 2x - 3$. Name a point in the shaded region. What does it mean for this point to lie in the shaded region?

2. Graph only $y \leq -x + 1$. Name a point in the shaded region. What does it mean for this point to lie in the shaded region?

3. The graph shows the two shaded regions on the same coordinate plane. Name a point in the overlapping region. What does it mean for this point to lie in the overlapping region?



THINK AND DISCUSS

4. **Describe** the steps you must take to find the solution region of a system of inequalities on a graphing calculator.

5. **Explain** how the solution region would change if the inequalities were $y \leq 2x - 3$ and $y \leq -x + 1$.

EXPLORATION

3-4 Linear Programming

Fred has two summer jobs. He can earn $15 per hour doing yard work and $10 per hour working at the mall. Each week, he must work less than 40 hours but earn at least $475.

1. Complete the table to determine if each combination satisfies Fred's criteria.

Plan	Hours of Yard Work	Hours at the Mall	Wages from Yard Work	Wages from the Mall	Total Wages
A	10	25			
B	30	9			
C	20	15			
D	25	10			
E	30	7			

2. Find another combination of work hours that meets Fred's criteria.

3. Let x represent the number of hours Fred does yard work and let y represent the number of hours Fred works at the mall. Write an inequality that describes Fred's goal for his weekly income.

THINK AND DISCUSS

4. **Describe** another inequality you can write to represent one of Fred's criteria.

5. **Discuss** whether Fred could work only at the mall and still meet his goals.

EXPLORATION

3-5 Linear Equations in Three Dimensions

Pizza Simple sells three sizes of pizzas: small, medium, and large. It sells the small for $8, the medium for $10, and the large for $12.

1. The table shows the sales for the weekend, Friday through Sunday. Complete the table to find the total revenue for each day.

	Small Pizzas Sold	Medium Pizzas Sold	Large Pizzas Sold	Total Daily Revenue
Fri	37	103	195	
Sat	60	195	235	
Sun	52	131	210	

2. Write an expression for the total daily revenue R. Let x represent the number of small pizzas sold. Let y represent the number of medium pizzas sold. Let z represent the number of large pizzas sold.

3. Write an equation for the total daily revenue R if the restaurant sells exactly 40 small pizzas.

THINK AND DISCUSS

4. **Explain** how you can use your equation to find the number of small pizzas sold if you know that 212 medium pizzas and 213 large pizzas were sold and that the total revenue was $5476.

5. **Determine** two combinations of pizza sales that would result in a revenue of $3600.

EXPLORATION

3-6 Solving Linear Systems in Three Variables

JetCo builds planes for major airlines. All of its planes have three sections: first class, business class, and coach.

Plane	Rows in First Class	Rows in Business Class	Rows in Coach	Total Seats
J101	3	4	12	168
J200	5	5	20	270
J444	5	7	25	337

1. All JetCo planes have x seats per row in first class, y seats per row in business class, and z seats per row in coach. For each plane, write an equation that represents the relationship between x, y, and z.
2. What system of equations would you need to solve in order to find the number of seats per row in each section?

THINK AND DISCUSS

3. **Discuss** whether the following advertisement is true: "All JetCo planes have 4 seats per row in first class, 9 seats per row in business class, and 10 seats per row in coach."
4. **Determine** the number of seats per row in coach, given that the planes have 4 seats per row in first class and 6 seats per row in business class.

4-1 Matrices and Data

A car company has three plants: A, B, and C. The company produces three cars: the Zip, the Tip, and the Pip.

The table shows the production numbers for each model at each plant during the first half of the year.

	Zip	Tip	Pip
A	2000	4200	7000
B	1900	4400	6200
C	450	1200	2500

1. Which plant produced the most cars?
2. For which model were the most cars produced?
3. The production numbers for the second half of the year are shown in the table.

	Zip	Tip	Pip
A	2100	3900	6700
B	2000	1200	6000
C	430	1050	2500

Make a new table that gives the production numbers for the entire year.

	Zip	Tip	Pip
A			
B			
C			

THINK AND DISCUSS

4. **Explain** how you found the entries in the table that gives the production numbers for the entire year.
5. **Describe** the dimensions of the table you would use to display data about a car company with four plants and six different models.

EXPLORATION

4-2 Multiplying Matrices

Three students sold T-shirts for a school fund-raiser. Short-sleeve and long-sleeve shirts were available. The table shows the quantities sold by each student.

	Short Sleeve	Long Sleeve
Colin	20	15
Sue	8	6
Lashonda	32	14

1. Create a matrix *A* for the quantity data. What are the dimensions of the matrix?

2. The table shows the prices for the two types of T-shirts. Create a matrix *B* for the pricing data. What are the dimensions of the matrix?

	Price ($)
Short Sleeve	10
Long Sleeve	15

3. Complete this table to show the total revenue that was brought in by each of the students.

	Total Revenue ($)
Colin	
Sue	
Lashonda	

4. Create a matrix *C* for the revenue data. What are the dimensions of the matrix?

THINK AND DISCUSS

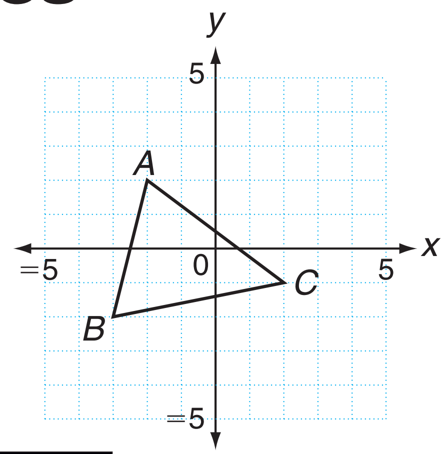
5. **Explain** how you completed the revenue table.

6. **Describe** how the dimensions of matrix *C* are related to the dimensions of matrices *A* and *B*.

EXPLORATION

4-3 Using Matrices to Transform Geometric Figures

You can use matrices to describe figures in the coordinate plane.



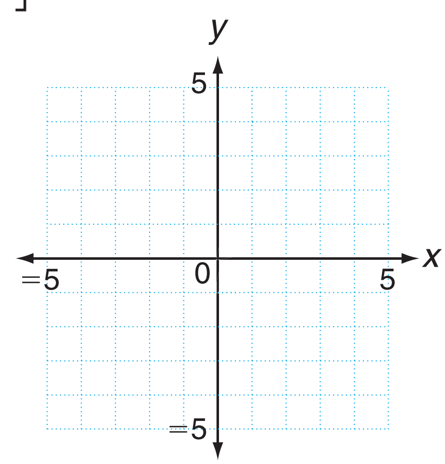
1. Complete the table by finding the coordinates of the vertices of $\triangle ABC$.

	Point *A*	Point *B*	Point *C*
x-coordinate			
y-coordinate			

2. Create a matrix *P* based on the data in the table.

3. Find the sum $P + R$ when $R = \begin{bmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix}$

4. Use the columns of the matrix you found in Problem 3 as the coordinates of the vertices of a triangle $\triangle A'B'C'$. Plot the vertices of $\triangle A'B'C'$ to graph the triangle.



THINK AND DISCUSS

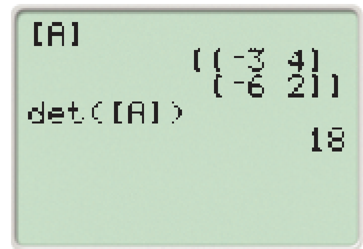
5. Describe how $\triangle A'B'C'$ is related to $\triangle ABC$.

6. Discuss what would have happened if you have added the matrix $\begin{bmatrix} 3 & 3 & 3 \\ -2 & -2 & -2 \end{bmatrix}$ to matrix *P*.

EXPLORATION

4-4 Determinants and Cramer's Rule

Every square matrix has an associated value called its *determinant*. You can use a calculator to find this value.



To find the determinant of a square matrix that you have already entered into your calculator, press 2nd MATRX x^{-1}, scroll to the right to **MATH**, and select **1:det(**. Then enter the name of the matrix by pressing 2nd MATRX x^{-1} and selecting the matrix from the list.
Close the parentheses and press ENTER.

Use your calculator to find the determinant of each matrix. Look for patterns.

1. $\begin{bmatrix} 3 & 0 \\ 0 & 10 \end{bmatrix}$

2. $\begin{bmatrix} 3 & 1 \\ 5 & 10 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 2 \\ 5 & 10 \end{bmatrix}$

4. $\begin{bmatrix} 2 & 3 \\ 10 & 5 \end{bmatrix}$

5. $\begin{bmatrix} 2 & 1 \\ 10 & 5 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 3 \\ -2 & 10 \end{bmatrix}$

THINK AND DISCUSS

7. **Describe** how the determinant is related to the entries of a 2×2 matrix.

8. **Explain** how you can find the determinant of $\begin{bmatrix} 2 & 1 \\ 10 & 5 \end{bmatrix}$ without using a calculator.

EXPLORATION

4-5 Matrix Inverses and Solving Systems

Use a calculator or paper and pencil for this Exploration.

1. Find the products *AB* and *BA* for the following matrices.

$$A = \begin{bmatrix} -2 & 0 \\ 5 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -0.5 & 0 \\ 2.5 & 1 \end{bmatrix}$$

2. Find the products *PQ* and *QP* for the following matrices.

$$P = \begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix} \qquad Q = \begin{bmatrix} 2 & -2.5 \\ -3 & 4 \end{bmatrix}$$

3. Find the products *ST* and *TS* for the following matrices.

$$S = \begin{bmatrix} 3 & 1 \\ 11 & 4 \end{bmatrix} \qquad T = \begin{bmatrix} 4 & -1 \\ -11 & 3 \end{bmatrix}$$

4. What do you notice about the products of the matrices in Problems 1–3?

THINK AND DISCUSS

5. **Explain** what is special about the matrix that you found to be the product in Problems 1–3.

6. **Describe** how the pairs of matrices you multiplied are similar to the numbers $\frac{3}{5}$ and $\frac{5}{3}$.

EXPLORATION

4-6 Row Operations and Augmented Matrices

You can use matrices to keep track of the steps used to solve a system of equations. The *augmented matrix* below contains the coefficients and constant terms of the system of equations.

$$\left[\begin{array}{cc|c} 4 & 1 & 14 \\ 2 & -5 & 18 \end{array}\right] \longrightarrow \begin{cases} 4x + y = 14 \\ 2x - 5y = 18 \end{cases}$$

Write each augmented matrix used to solve the system of equations above. Write the row operation that was used to form it from the previous augmented matrix (Step 1 is done for you).

	System		Augmented Matrix	Row Operation
1.	$\begin{cases} 4x + y = 14 \\ 4x - 10y = 36 \end{cases}$	$\longrightarrow$	$\left[\begin{array}{cc\|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$	Multiply the second row by 2.
2.	$\begin{cases} 4x + y = 14 \\ y = 22 \end{cases}$	$\longrightarrow$	$\left[\begin{array}{cc\|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$	
3.	$\begin{cases} 4x + y = 14 \\ y = -2 \end{cases}$	$\longrightarrow$	$\left[\begin{array}{cc\|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$	
4.	$\begin{cases} 4x = 16 \\ y = -2 \end{cases}$	$\longrightarrow$	$\left[\begin{array}{cc\|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$	
5.	$\begin{cases} x = 4 \\ y = -2 \end{cases}$	$\longrightarrow$	$\left[\begin{array}{cc\|c} \square & \square & \square \\ \square & \square & \square \end{array}\right]$	

THINK AND DISCUSS

6. **Describe** what you notice about the 2×2 matrix on the left side of the augmented matrix in the final step of the solution.

7. **Describe** what you notice about the 2×1 matrix on the right side of the augmented matrix in the final step of the solution.

5-1 Using Transformations to Graph Quadratic Functions

The simplest quadratic function is $f(x) = x^2$. You can plot points to explore the graphs of quadratic functions.

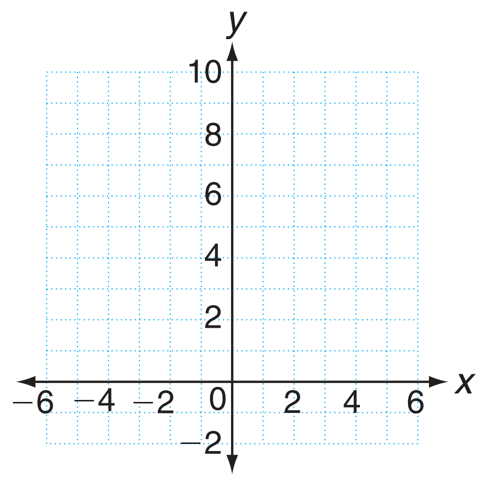
1. Complete the table for the function $f(x) = x^2$.

x	−3	−2	−1	0	1	2	3
$f(x)$							

2. Graph the function f by plotting the ordered pairs on the coordinate plane.

3. Complete the table for the function $g(x) = (x - 1)^2$.

x	−3	−2	−1	0	1	2	3
$g(x)$							



4. Graph the function g by plotting the ordered pairs on the coordinate plane.

THINK AND DISCUSS

5. **Explain** how the graph of g is related to the graph of f.

6. **Describe** the graph of $h(x) = (x - 2)^2$ as a transformation of the graph of f.

EXPLORATION

5-2 Properties of Quadratic Functions in Standard Form

You can use a calculator to explore properties of the graphs of quadratic functions. To graph a quadratic function, press Y= and enter the function rule. Then press GRAPH to see the graph.

1. Use your calculator to graph each function listed below. Then complete the table.

Function	Parabola Opens Upward or Downward?	*y*-intercept
$f(x) = 2x^2 + 4x + 1$		
$f(x) = -3x^2 - 6x + 2$		
$f(x) = x^2 + 2x - 4$		
$f(x) = -x^2 - 3x - 5$		

2. The quadratic functions in Problem 1 are written in the form $f(x) = ax^2 + bx + c$. For each function, write the values of a and c.

Function	Value of a	Value of c
$f(x) = 2x^2 + 4x + 1$		
$f(x) = -3x^2 - 6x + 2$		
$f(x) = x^2 + 2x - 4$		
$f(x) = -x^2 - 3x - 5$		

THINK AND DISCUSS

3. **Explain** how the sign of a can help you determine whether the graph of a quadratic function opens upward or downward.

4. **Explain** the relationship between the value of c and the y-intercept of a quadratic function.

EXPLORATION

5-3 Solving Quadratic Equations by Graphing and Factoring

The zero of a function is a value of the input x that makes the output $f(x)$ equal to zero. The factored form of a quadratic function can help you determine its zeros.

1. Complete the table for the function $f(x) = (x - 3)(x + 2)$.

x	-3	-2	-1	0	1	2	3
$f(x)$							

2. What are the zeros of the function f?

3. Complete the table for the function $g(x) = (x - 2)(x + 1)$.

x	-3	-2	-1	0	1	2	3
$g(x)$							

4. What are the zeros of the function g?

THINK AND DISCUSS

5. **Explain** the relationship between the zeros of a quadratic function and the factored form of the function's rule.

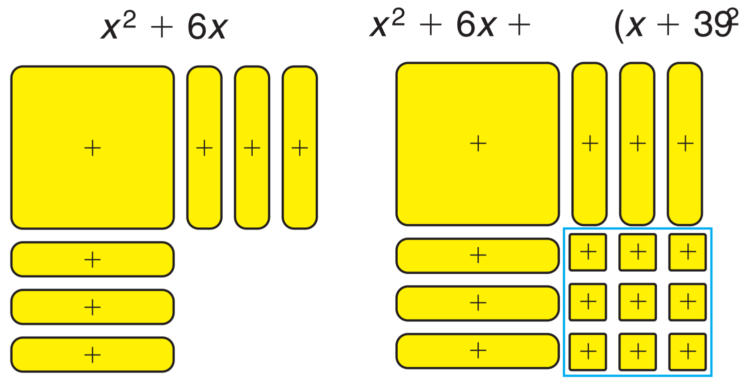
6. **Describe** how to find the zeros of the function $h(x) = (x - 1)(x + 2)$ without making a table or graph.

EXPLORATION

5-4 Completing the Square

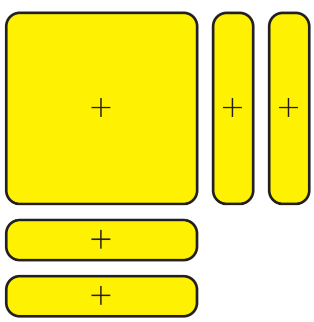
You can add a term to a quadratic expression of the form $x^2 + bx$ to form a perfect square trinomial. This is called completing the square.

The model shows completing the square for $x^2 + 6x$ by adding 9 unit tiles. The perfect-square trinomial that results is $x^2 + 6x + 9 = (x + 3)^2$.

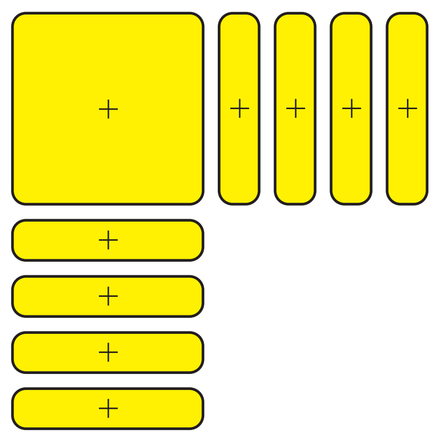


Complete the square for each model by adding unit tiles. Then write the perfect square trinomial that results.

1. $x^2 + 4x$



2. $x^2 + \quad x$



THINK AND DISCUSS

3. **Tell** how you can complete the square for $x^2 + 12x$ without using a model.

EXPLORATION

5-5 Complex Numbers and Roots

Recall that the Product Property of Square Roots states that $\sqrt{ab} = \sqrt{a} : \sqrt{b}$. Use this property to write each expression as the product of an integer and $\sqrt{-1}$.

1. $\sqrt{-36}$
2. $-\sqrt{-4}$
3. $\sqrt{-81}$
4. $-\sqrt{-144}$

The number $\sqrt{-1}$ is often represented by *i.* For example, $\sqrt{-25}$ can be written as $\sqrt{25} \cdot \sqrt{-1}$, or 5*i*. Write each expression in terms of *i*.

5. $\sqrt{-36}$
6. $-\sqrt{-4}$
7. $\sqrt{-81}$
8. $-\sqrt{-144}$

THINK AND DISCUSS

9. **Explain** why the square root of any negative number can be written in terms of *i*.

10. **Discuss** how *i* can be used to express the solutions of the quadratic equation $x^2 + 4 = 0$.

EXPLORATION

5-6 The Quadratic Formula

A quadratic equation may have two real solutions, one real solution, or two nonreal complex solutions. For a quadratic equation of the form $ax^2 + bx + c = 0$, you can use the values of a, b, and c to determine the type and number of solutions.

1. Complete the table. Use any method to solve each quadratic equation, and then use its values of a, b, and c to evaluate the expression $b^2 - 4ac$.

Equation	Solutions	Value of $b^2 - 4ac$
$x^2 + 5x + 6 = 0$		
$x^2 + 2x + 1 = 0$		
$x^2 + 4 = 0$		
$x^2 - 6x + 9 = 0$		
$x^2 + 10 = 0$		
$x^2 + 3x - 4 = 0$		

2. Based on the table, what type and number of solutions does a quadratic equation have if the value of $b^2 - 4ac$ is 0?
3. What type and number of solutions does a quadratic equation have if the value of $b^2 - 4ac$ is less than 0?
4. What type and number of solutions does a quadratic equation have if the value of $b^2 - 4ac$ is greater than 0?

THINK AND DISCUSS

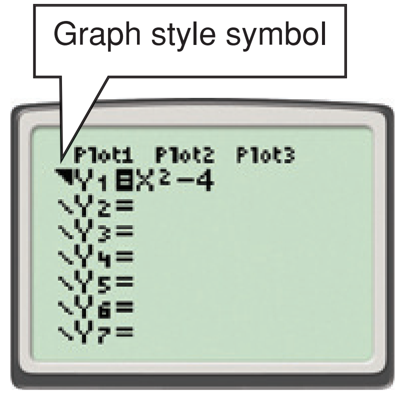
5. **Describe** how you can determine the type and number of solutions of $x^2 + 4x - 30 = 0$ without solving the equation.

EXPLORATION

5-7 Solving Quadratic Inequalities

You can use a graphing calculator to explore quadratic inequalities.

Graph the inequality $y \geq x^2 - 4$ as follows. Press Y= and enter $x^2 - 4$ for **Y1**. Then use the arrow keys to move the cursor to the left of **Y1**. Press ENTER until the graph style changes to the symbol shown. This symbol indicates that the area above the graph will be shaded. Press GRAPH to view the graph.



1. Describe the graph of the inequality $y \geq x^2 - 4$.

2. The shaded area of the graph represents the solution set of the inequality. Tell whether each of the following points is in the solution set of $y \geq x^2 - 4$.

   a. (0, 0) b. (3, 0)

   c. (1, 6) d. (−3, −3)

3. What are the possible values of y for $x = 0$?

4. What is the least possible value of y?

THINK AND DISCUSS

5. **Discuss** how the graph of $y \leq x^2 - 4$ would differ from the graph of $y \geq x^2 - 4$.

EXPLORATION

5-8 Curve Fitting with Quadratic Models

You can use differences to analyze patterns in data. Complete each table by first finding the missing *y*-values. Subtract consecutive *y*-values to find first differences. Then subtract consecutive first differences to find second differences.

1. $y = x^2 + 1$

x	−3	−2	−1	0	1
y	10	5	2		
First Differences	−5	−3			
Second Differences	2				

2. $y = 3x^2 + x + 2$

x	−1	0	1	2	3
y					
First Differences					
Second Differences					

3. $y = 5x^2 - 2x$

x	−4	−3	−2	−1	0
y					
First Differences					
Second Differences					

THINK AND DISCUSS

4. **Discuss** any patterns you notice in the second differences of the tables.

5. **Explain** how you can determine whether a set of data can be represented by a quadratic function.

EXPLORATION

5-9 Operations with Complex Numbers

Recall that $i = \sqrt{-1}$ and that $i^2 = -1$. You can use these facts to simplify other powers of i. For example, $i^3 = i^2 \cdot i = -1 \cdot i = -i$.

1. Complete the table by simplifying the powers of i.

$i^1 =$	$i^2 =$	$i^3 =$	$i^4 =$
$i^5 =$	$i^6 =$	$i^7 =$	$i^8 =$
$i^9 =$	$i^{10} =$	$i^{11} =$	$i^{12} =$
$i^{13} =$	$i^{14} =$	$i^{15} =$	$i^{16} =$

2. What values are possible for the positive integer powers of i?

THINK AND DISCUSS

3. **Discuss** the pattern you notice in the table.

4. **Explain** how you can quickly find the value of i^{64}.

EXPLORATION

6-1 Polynomials

The table shows examples of expressions that are polynomials and examples of expressions that are not polynomials.

Polynomials	Not Polynomials
$5x^2$	$5x^{-2}$
$3y^7 - 4y^2 + 1$	$3y^{-7} - 4y^2 + 1$
$\frac{1}{3}x^4$	$x^{1/3}$
$\sqrt{7}t$	$7\sqrt{t}$
$0.13a^3 + 5.12a^2$	$0.13a^{0.4}$
$-m + 1$	$\lvert m + 1 \rvert$
$\frac{6}{7}y^8$	$\frac{6}{7y^8}$
$7x - 5$	$7^x - 5$

Use the examples to help you decide whether each of the following is a polynomial.

1. $14x^5 - 4x^2 + x$
2. $-3y^2 + y^{-2}$
3. $m^{0.5} - m^{12}$
4. $3^{1/2} \cdot x - 6$
5. What do you notice about the exponents of the variables in the expressions that are polynomials?

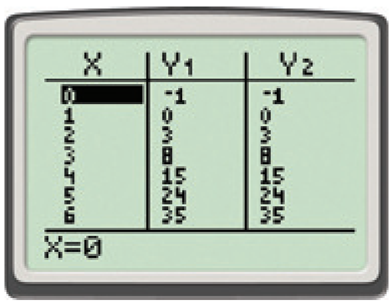
THINK AND DISCUSS

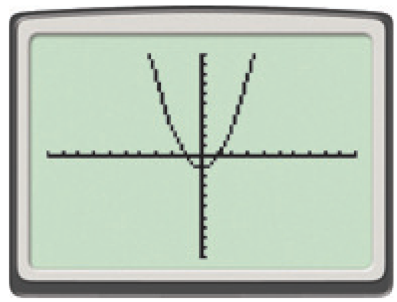
6. **Describe** the characteristics of expressions that are polynomials.
7. **Explain** why $-k^5 + 3q^4 + 8w^{0.75}$ is not a polynomial.

EXPLORATION

6-2 Multiplying Polynomials

You can use a calculator to check whether you have correctly multiplied polynomials. For example, to verify that $(x + 1)(x - 1)$ is equal to $x^2 - 1$, enter the expressions as Y1 and Y2. The table shows that the two expressions have the same value for all the listed values of x. In addition, the graphs of the corresponding functions appear to coincide.





Use a calculator to determine whether each multiplication was performed correctly.

1. $5x^2(2x^2 + 3) = 10x^4 + 3$
2. $(3x^2 - 2)(x - 7) = 3x^3 - 21x^2 - 2x + 14$
3. $-4x^3(2x - 3) = -8x^4 + 12x^3$
4. $(3x + 1)(4x^3 - 3x^2 + 1) = 12x^4 - 9x^3 + 3x$

THINK AND DISCUSS

5. **Explain** how to recognize that certain expressions are not equivalent.
6. **Explain** how to multiply a monomial and a polynomial.

EXPLORATION

6-3 Dividing Polynomials

To divide polynomials, you can use the same method as that for dividing numbers.

1. Use arithmetic long division to divide $693 \div 21$.

2. Use polynomial long division to divide $(6x^2 + 9x + 3) \div (2x + 1)$.

3. Compare your answers in Problems 1 and 2. What can you say about the coefficients of the quotients for each?

4. Use your answer to Problem 3 to make a conjecture about the quotient of $(x^2 + 6x + 8) \div (x + 4)$ based on the quotient of $168 \div 14$. Verify your conjecture by using polynomial long division.

THINK AND DISCUSS

5. Explain how you can find the quotient of $(x^2 + 6x + 5) \div (x + 5)$.

6. Discuss whether every polynomial long division problem matches exactly with a numerical long division problem. Give an example to support your response.

EXPLORATION

6-4 Factoring Polynomials

Recall from the Remainder Theorem that if a polynomial function $P(x)$ is divided by $x - a$, then the remainder is $P(a)$.

1. Use synthetic division to find the remainder when the polynomial $P(x) = x^3 - 3x^2 + x - 3$ is divided by $x - 3$.

2. What is $P(3)$?

3. Use the results of the synthetic division in Problem 1 to write $P(x)$ as the product of $x - 3$ and another polynomial.

4. Use synthetic division to find the remainder when the polynomial $Q(x) = 2x^4 - 4x^3 + 4x^2 - 7x - 2$ is divided by $x - 2$.

5. What is $Q(2)$?

6. Use the results of the synthetic division in Problem 4 to write $Q(x)$ as the product of $x - 2$ and another polynomial.

THINK AND DISCUSS

7. **Describe** what you can conclude about $x - a$ if you know that $P(a) = 0$.

8. **Explain** why $x - 1$ must be a factor of $x^6 - 1$.

EXPLORATION

6-5 Finding Real Roots of Polynomial Equations

Recall that you can find the roots of the polynomial equation $P(x) = 0$ by setting each factor of $P(x)$ equal to zero and solving for x.

1. Complete the table. Write your answers in fraction form.

$P(x)$	Factored Form of $P(x)$	Roots of $P(x) = 0$
$12x^2 - 41x + 35$	$(3x - 5)(4x - 7)$	
$6x^2 - 5x - 4$	$(2x + 1)(3x - 4)$	
$10x^3 + 3x^2 - 31x + 6$	$(2x - 3)(5x - 1)(x + 2)$	
$48x^3 - 32x^2 - 27x + 18$	$(3x - 2)(4x + 3)(4x - 3)$	

2. Look for a pattern in the table. The numerators of the roots are factors of the coefficient of which term of the polynomial?

3. The denominators of the roots are factors of the coefficient of which term of the polynomial?

THINK AND DISCUSS

4. **Explain** how you might complete the following statement: If the polynomial $P(x)$ has integer coefficients and if $\frac{a}{b}$ is a root of $P(x) = 0$, then ______.

5. **Explain** how you know that $\frac{2}{7}$ is not a root of $5x^2 + x + 3 = 0$.

EXPLORATION

6-6 Fundamental Theorem of Algebra

In this Exploration you will make a conjecture about the number of roots of a polynomial function.

1. Complete the table. Include multiplicities when finding the number of roots.

$P(x)$	Factored Form of $P(x)$	Roots of $P(x) = 0$	Number of Roots
$x^2 - 16$			
$x^3 - 5x^2 + 8x - 4$	$(x - 1)(x - 2)^2$		
$x^5 - x^4 - 2x^3$			
$x^4 - 9x^2$			
$x^6 - x^5 - 16x^4 - 20x^3$	$x^3(x - 5)(x + 2)^2$		

2. Look for a pattern in your table. How is the number of roots related to $P(x)$?

THINK AND DISCUSS

3. **Discuss** how you can make a generalization based on your findings.

4. **Explain** how you can use your generalization to make a statement about the roots of $x^{10} - 5x^7 + 12x = 0$.

EXPLORATION

6-7 Investigating Graphs of Polynomial Functions

Use a calculator to explore the connection between the graph of a polynomial function and the degree of the polynomial.

Graph each of the following polynomial functions.

1. $f(x) = x^2 - x - 8$
2. $f(x) = 2x^3 - 3x + 4$
3. $f(x) = x^4 - 8x^2 + 8$
4. $f(x) = x^5 - 6x^3 + 5x - 3$
5. In general, what happens to the graph of a polynomial function as the degree of the polynomial increases?
6. Graph $f(x) = x^8 + 2x^2 + 1$. Does this graph fit the relationship you observed in Problem 2? Why or why not?

THINK AND DISCUSS

7. **Describe** how the graphs of the functions with odd degrees are different from the graphs of the functions with even degrees.
8. **Describe** what you would expect the graph of a 7th-degree polynomial function to look like.

EXPLORATION

6-8 Transforming Polynomial Functions

Use your calculator to investigate transformations of polynomial functions.

1. Graph $f(x) = x^3 - 6x + 2$.

Graph each of the following functions in the same window as $f(x)$. In each case, use the language of transformations to explain how the graph of $g(x)$ is related to the graph of $f(x)$.

2. $g(x) = x^3 - 6x + 5$

3. $g(x) = (x - 3)^3 - 6(x - 3) + 2$

4. $g(x) = \frac{1}{2}x^3 - 3x + 1$

5. $g(x) = \left(\frac{1}{3}x\right)^3 - 2x + 2$

6. $g(x) = -x^3 + 6x - 2$

THINK AND DISCUSS

7. **Explain** how you can write a polynomial function whose graph is the same as that of $f(x)$ but is translated 6 units down.

8. **Describe** how you can graph $Q(x) = (x + 2)^5 - 3(x + 2)^2$ if you know what the graph of $P(x) = x^5 - 3x^2$ looks like.

6-9 Curve Fitting with Polynomial Models

You can discover an interesting property of polynomial functions by investigating finite differences.

1. Complete the table by finding the *y*-values of the function $y = x^3 + 2x$ and calculating the first, second, and third differences of the *y*-values.

x	−2	−1	0	1	2	3
y						
First Differences						
Second Differences						
Third Differences						

2. Complete the table by finding the *y*-values of the function $y = x^4 - 3x$ and calculating the first, second, and third differences of the *y*-values.

x	−1	0	1	2	3	4	5
y							
First Differences							
Second Differences							
Third Differences							
Fourth Differences							

THINK AND DISCUSS

3. **Discuss** the relationship between finite differences and the degree of a polynomial function.
4. **Explain** what conclusions you can draw from a table of data for which the fifth differences are constant.

EXPLORATION

7-1 Exponential Functions, Growth, and Decay

A biologist is studying a type of cell that divides in two every hour. The biologist begins the experiment with a single cell. The population doubles every hour.

1. Complete the table.

Time (h)	0	1	2	3	4	5
Cells	1					

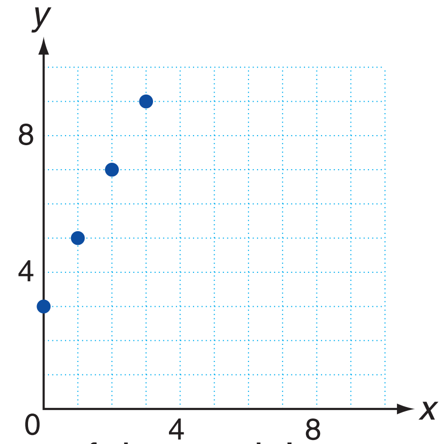
2. How many hours will it take until there are more than 500 cells?
3. How many cells will there be after 10 hours?
4. How many cells will there be after *n* hours?

THINK AND DISCUSS

5. **Explain** how you can write a function that models this situation.
6. **Describe** how your function would be different if the biologist started the experiment with 3 cells

7-2 Inverses of Relations and Functions

The graph shows the distance of a particle from a location over time.



1. Complete the table based on the data in the graph.

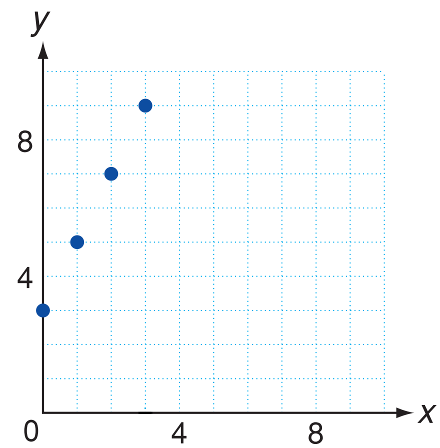
Time (s)	0	1	2	3
Distance (ft)				

2. Write an equation that gives the distance of the particle as a function of time.

3. Complete the table showing the time it takes the particle to reach a given distance.

Distance (ft)	3	5	7	9
Time (s)				

4. Plot the ordered pairs on the same coordinate plane as the original set of data.



THINK AND DISCUSS

5. **Discuss** how the two sets of data points in the graph are related to each other. (*Hint*: Consider the line $y = x$.)

6. **Explain** how you could use your equation to find the time it takes the particle to reach a given distance.

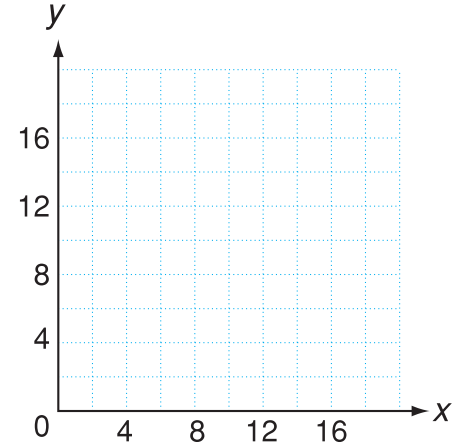
EXPLORATION

7-3 Logarithmic Functions

The population of a bacteria colony doubles every hour. The colony starts with one bacterium.

1. Complete the table.

Time	0	1	2	3	4
Population					

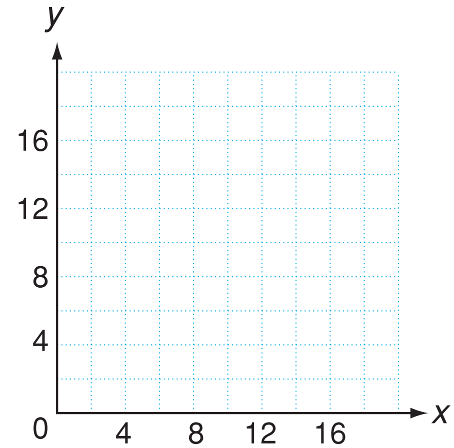


2. Plot the ordered pairs to show the population growth over the first 4 hours.

3. Write an equation that gives the population as a function of the time.

4. Complete this table to show the inverse relationship between time and population.

Population					
Time	0	1	2	3	4



5. Plot the ordered pairs from this table on the same coordinate plane.

THINK AND DISCUSS

6. Discuss the relationship between the graphs of the two sets of data points.

7. Explain what you would need to do in order to use your equation from Problem 3 to write a new equation that gives the time as a function of the population.

EXPLORATION

7-4 Properties of Logarithms

You can discover properties of logarithms by comparing logarithmic expressions.

1. Evaluate the expressions in the table.

$\log_2(4 \cdot 8) = \log_2 32 = $ ___	$\log_2 4 + \log_2 8 = $ ___
$\log_2(2 \cdot 32) = \log_2 64 = $ ___	$\log_2 2 + \log_2 32 = $ ___
$\log_2(1 \cdot 8) = \log_2 8 = $ ___	$\log_2 1 + \log_2 8 = $ ___

2. What do you notice?

3. Evaluate the expressions in the table.

$\log_2\left(\frac{64}{16}\right) = \log_2 4 = $ ___	$\log_2 64 - \log_2 16 = $ ___
$\log_2\left(\frac{16}{2}\right) = \log_2 8 = $ ___	$\log_2 16 - \log_2 2 = $ ___
$\log_2\left(\frac{256}{8}\right) = \log_2 32 = $ ___	$\log_2 256 - \log_2 8 = $ ___

4. What do you notice?

THINK AND DISCUSS

5. **Make** a conjecture about the expression $\log_b(mn)$.

6. **Make** a conjecture about the expression $\log_b\left(\frac{m}{n}\right)$.

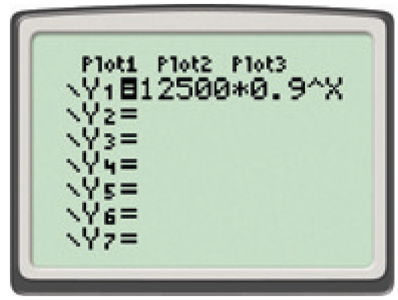
EXPLORATION

7-5 Exponential and Logarithmic Equations and Inequalities

To predict the number of prairie dogs in a region, a scientist uses the equation $P = 12{,}500(0.9)^t$ where P is the population after t years.

1. Based on the equation, is the population of prairie dogs increasing or decreasing? How do you know?
2. Find the initial population at time $t = 0$.
3. Enter the function into your calculator as **Y1**. Press 2nd WINDOW (UBLSEU) to set **TblStart=0** and **ΔTbl=1**.

   Then press 2nd GRAPH (UABLE) to make a table for the function.



4. After how many years will the population of prairie dogs be 10,125?
5. After how many years will the population of prairie dogs fall below 6000?

THINK AND DISCUSS

6. **Explain** how you could write an equation that represents Problem 4.
7. **Explain** how you could write an inequality that represents Problem 5.

EXPLORATION

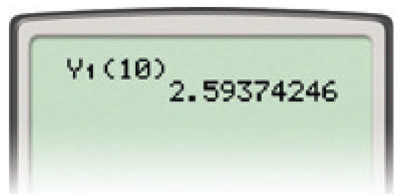
7-6 The Natural Base, *e*

In this Exploration, you will investigate the expression $\left(1 + \frac{1}{n}\right)^n$.

1. Evaluate $\left(1 + \frac{1}{n}\right)^n$ for $n = 1$.

2. Evaluate $\left(1 + \frac{1}{n}\right)^n$ for $n = 2$.

3. You can use your calculator to help you evaluate the expression for larger values of *n*. First enter the expression as the function **Y1**. Then press 2nd Q IU MODE to return to the home screen. Press VARS, scroll right to **Y-VARS**, select **1:Function**, and choose **1:Y1**. Now you can evaluate the function for any input by entering a value in parentheses as shown.



4. Use your calculator to evaluate the expression for $n = 10{,}000$, $n = 100{,}000$, and $n = 1{,}000{,}000$.

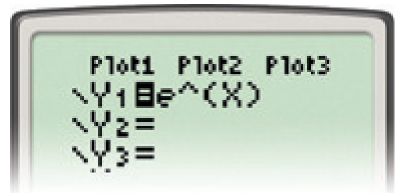
THINK AND DISCUSS

5. **Describe** what happens to the value of the expression as *n* gets larger.

6. **Explain** what you think the graph of the function $f(n) = \left(1 + \frac{1}{n}\right)^n$ would look like.

7-7 Transforming Exponential and Logarithmic Functions

Use a calculator to explore transformations of exponential functions.



1. Enter the parent function $y = e^x$ by pressing [Y=] and then [2nd] [LN] (e^x) [X,T,θ,n] [ )]. Graph the function.
2. Enter each of the following functions as **Y2** and graph them in the same window as the graph of $y = e^x$. In each case, describe how the graph of the function is related to that of $y = e^x$.
   a. $y = e^x - 5$
   b. $y = e^{x-4}$
   c. $y = -e^x$
   d. $y = e^{-x}$

THINK AND DISCUSS

3. **Explain** how the graph of $y = e^{x-1} + 2$ is related to the graph of $y = e^x$.
4. **Describe** how you would change the equation $y = 4^x$ if you wanted to translate the graph of this function 3 units to the left.

EXPLORATION

7-8 Curve Fitting with Exponential and Logarithmic Models

Two scientists use two different models to predict populations of elk in a national forest.

Model 1	
Year t	Population p
0	32,000
1	33,500
2	35,000
3	36,500

Model 2	
Year t	Population p
0	32,000
1	33,600
2	35,280
3	37,044

1. For model 1, is p a linear function of t, a quadratic function of t, or neither? How do you know?
2. For model 2, is p a linear function of t, a quadratic function of t, or neither? How do you know?
3. For model 2, find the ratio of each y-value to the previous one. What do you notice?
4. Describe how the population of elk increases in model 2.

THINK AND DISCUSS

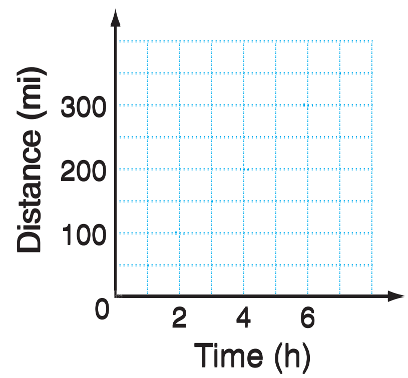
5. **Explain** what type of function is used to model the elk population in model 2.
6. **Discuss** how you can recognize an exponential function from a table of ordered pairs.

8-1 Variation Functions

Distance traveled *d* can be determined by using the formula $d = rt$, where *r* is the rate of travel and *t* is the time traveled.

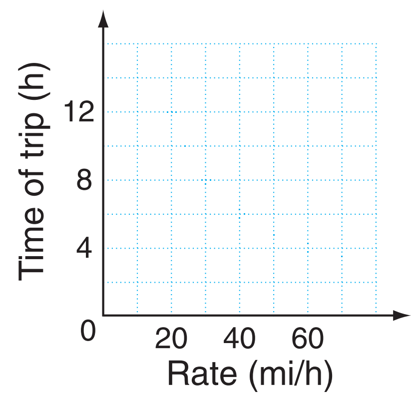
1. Jorge's averages 50 mi/h during a driving trip. Use this information to complete the table. Then graph the data.

Time (h)	Distance (mi)
0	
2	
4	
6	



2. Leanne is planning to drive 240 miles. Use this information to complete the table. Then graph the data.

Rate (mi/h)	Time of Trip (h)
20	
30	
40	
60	



THINK AND DISCUSS

3. Tell how the graphs in Problems 1 and 2 are different.

4. Describe how the amount of time Jorge travels affects the distance he travels.

5. Describe how Leanne's rate, or speed, affects the amount of time she needs for her trip.

EXPLORATION

8-2 Multiplying and Dividing Rational Expressions

A *rational expression* is a quotient of two polynomials. You can multiply and divide rational expressions much like you would multiply and divide fractions.

Find each product. Simplify if necessary.

1. $\frac{1}{3} \cdot \frac{2}{3}$

2. $\frac{3}{10} \cdot \frac{5}{6}$

3. List the steps you used to multiply the fractions.

Find each product by using the steps you listed in Problem 3. Assume that $x \neq 0$.

4. $\frac{2}{5} \cdot \frac{3}{x}$

5. $\frac{x}{8} \cdot \frac{y}{x}$

Find each quotient. Simplify if necessary.

6. $\frac{7}{10} \div \frac{1}{2}$

7. $\frac{3}{4} \div \frac{9}{16}$

8. List the steps you used to divide the fractions.

Find each quotient by using the steps you listed in Problem 8. Assume that $x \neq 0$.

9. $\frac{4}{x} \div \frac{2}{3}$

10. $\frac{y}{x} \div \frac{3}{x}$

THINK AND DISCUSS

11. **Discuss** how to multiply and divide rational expressions.

EXPLORATION

8-3 Adding and Subtracting Rational Expressions

You can add and subtract rational expressions much like you would add and subtract fractions.

Find each sum. Simplify if necessary.

1. $\frac{3}{5} + \frac{4}{5}$

2. $\frac{7}{10} + \frac{5}{10}$

3. List the steps you used to add the fractions.

Find each sum by using the steps you listed in Problem 3. Assume that $x \neq 0$.

4. $\frac{2}{x} + \frac{5}{x}$

5. $\frac{4}{3x} + \frac{8}{3x}$

Find each difference. Simplify if necessary.

6. $\frac{5}{8} - \frac{3}{8}$

7. $\frac{4}{12} - \frac{8}{12}$

8. List the steps you used to subtract the fractions.

Find each difference by using the steps you listed in Problem 8. Assume that $x \neq 0$.

9. $\frac{6}{4x} - \frac{1}{4x}$

10. $\frac{12}{5x} - \frac{7}{5x}$

THINK AND DISCUSS

11. **Discuss** how to add and subtract rational expressions with like denominators.

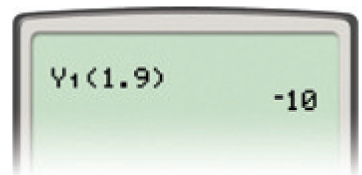
EXPLORATION

8-4 Rational Functions

You can explore the behavior of the rational function $f(x) = \frac{1}{x - 2}$ by using a graphing calculator.

1. Enter the function rule by pressing Y= and entering **1/(X − 2)**. Graph the function by pressing ZOOM and selecting **4:ZDecimal.** Describe the function's graph.

Select the function you entered by pressing VARS, scrolling right to **Y-VARS**, selecting **1:Function**, and choosing **1:Y1**. You can evaluate the function for any *x*-value by entering a value in parentheses as shown.



2. Complete the tables for the function $f(x) = \frac{1}{x - 2}$.

x	*y*
1.9	
1.99	
1.999	

x	*y*
2.1	
2.01	
2.001	

THINK AND DISCUSS

3. **Explain** what happens to the value of *y* as the value of *x* gets closer and closer to 2.
4. **Discuss** why the function $f(x) = \frac{1}{x - 2}$ is undefined at $x = 2$.

EXPLORATION

8-5 Solving Rational Equations and Inequalities

A rancher wants to build a rectangular holding pen.

1. The pen will have an area of 450 ft^2. Let *w* represent the width of the pen. Write an expression in terms of *w* for the length of the pen.

2. The length of the pen will be twice the width. Use this information to write a second expression in terms of *w* for the length of the pen.

3. Write an equation by setting the two expressions you wrote for the length of the pen equal to each other.

4. Solve the equation for *w*.

5. What will be the dimensions of the holding pen?

THINK AND DISCUSS

6. **Describe** the steps you used to solve the equation in Problem 4.

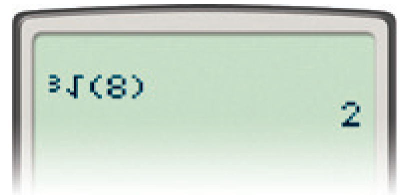
7. **Explain** how you could check your solution to the equation.

8. **Tell** how you knew which value of *w* to use for the width of the pen.

EXPLORATION

8-6 Radical Expressions and Rational Exponents

You can use a graphing calculator to investigate the meaning of cube roots. To enter cube roots, press MATH and select **4:** $\sqrt[3]{}$**(**.



1. Use your calculator to help you complete the table.

Cubes	Cube Roots
$1^3 =$	$\sqrt[3]{1} =$
$2^3 =$	$\sqrt[3]{8} =$
$3^3 =$	$\sqrt[3]{27} =$
$4^3 =$	$\sqrt[3]{64} =$
$5^3 =$	$\sqrt[3]{125} =$

2. Based on the pattern in the table, what is the cube root of 6^3?

THINK AND DISCUSS

3. Explain what is meant by the cube root of a real number *a*.

4. Discuss the meaning of the expression $\sqrt[4]{16}$ and how you could determine its value.

EXPLORATION

8-7 Radical Functions

You can explore the behavior of radical functions by using a graphing calculator.

1. Enter the function $f(x) = \sqrt{x}$ by pressing **Y=** and entering $\sqrt{x}$. Graph the function in the standard square window by pressing **ZOOM** and selecting **6:ZStandard** and by pressing **ZOOM** again and selecting **5:ZSquare**. Based on the graph, what is the domain of the function $f(x) = \sqrt{x}$?

2. Why is the function f restricted to this domain?

Graph each of the following functions on a graphing calculator and give its domain.

3. $f(x) = \sqrt{x - 2}$

4. $f(x) = \sqrt{x - 5}$

5. $f(x) = \sqrt{x + 4}$

6. $f(x) = \sqrt{x + 1}$

THINK AND DISCUSS

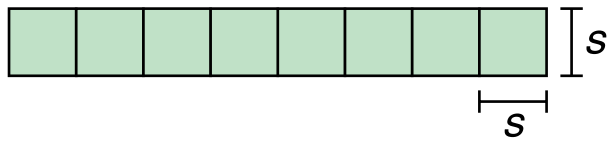
7. **Describe** how the graph of $y = \sqrt{x - h}$ is related to the graph of $y = \sqrt{x}$.

8. **Explain** how you can determine the domain of the function $y = \sqrt{x - h}$.

EXPLORATION

8-8 Solving Radical Equations and Inequalities

An architect is designing the layout of an office building. One section of the building will consist of a row of 8 identical offices, each with a square-shaped floor.



1. Write an equation that represents the total area *A* of the offices in terms of *s*, the side length of each office.
2. Solve your equation from Problem 1 for *s*.
3. Use your calculator to help you complete the table.

Total Area (ft^2)	Office Side Length (ft)
882	
968	
1058	
1152	
1250	

4. For what values of *A* does each office have a side length of at least 12 ft?

THINK AND DISCUSS

5. **Give** two equations you could use to determine the total area of the offices when given that each office has a side length of 14 ft.

6. **Explain** the steps you would use to solve each equation.

EXPLORATION

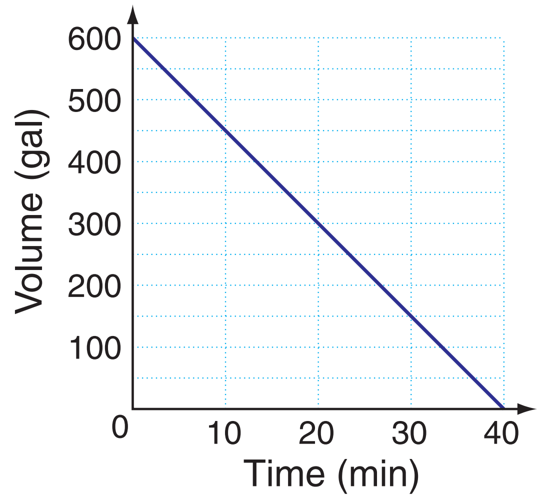
9-1 Multiple Representations of Functions

Many real-world situations can be represented with a verbal description, a graph, a table, and an equation.

1. Complete the table for the graph shown here.

Time (min)	Volume (gal)
0	
10	
20	
30	
40	

Volume of Water in Tank



2. Write a verbal description of the situation shown on the graph.

3. How many gallons are drained from the tank every 10 minutes? How many gallons are drained every minute?

4. Write an equation for the situation, where x is the time in minutes and y is the volume of water in the tank in gallons.

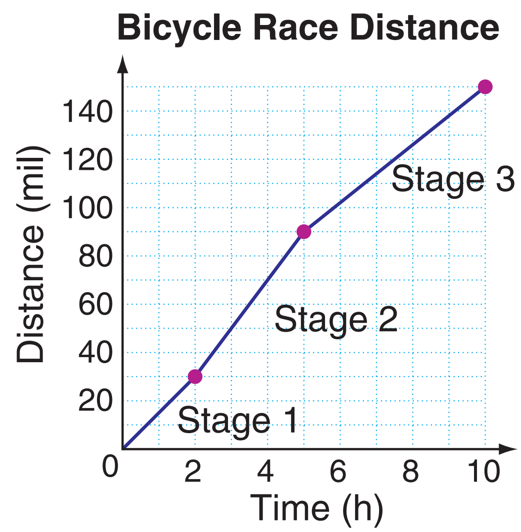
THINK AND DISCUSS

5. **Discuss** the advantages and disadvantages of using a graph to represent the situation.

6. **Discuss** the advantages and disadvantages of using an equation to represent the situation.

9-2 Piecewise Functions

The graph shows a 10-hour training session for a bicycle racer. The session consists of three different stages.



1. Write a verbal description of the training session.
2. What is the cyclist's speed during each stage of the training session?
3. Write a linear equation for each stage of the training session, where *x* represents the time in hours and *y* represents the cyclist's distance in miles. (*Hint*: Use the point-slope form for the equation of a line.)
4. What is the domain for each stage of the training session?

THINK AND DISCUSS

5. **Explain** how you can use only the graph to determine the stage during which the cyclist had the greatest speed.
6. **Describe** how the graph would have been different if the cyclist had maintained the same speed during all three stages.

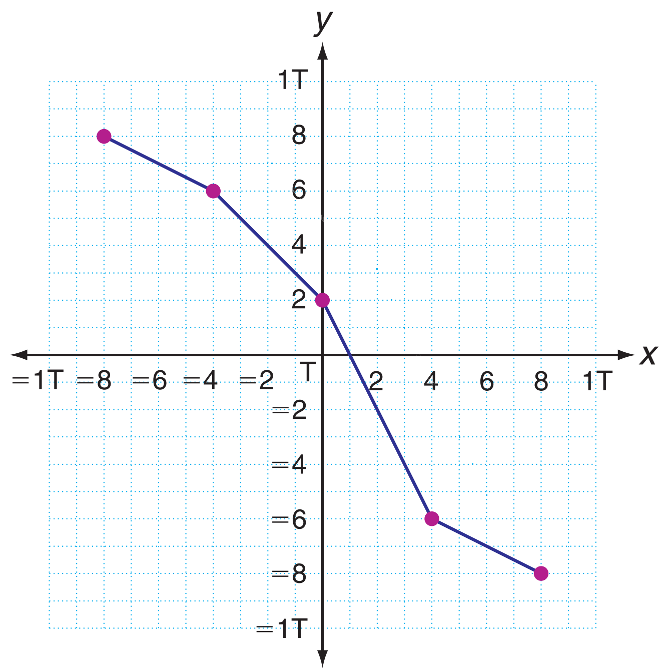
EXPLORATION

9-3 Transforming Functions

In this Exploration, you will investigate a transformation of a piecewise function.

1. Complete the table for the piecewise function in the graph.

x	y
−8	
−4	
0	
4	
8	



2. Add 2 to each x-value in the table and add 1 to each y-value. Plot the new points and connect them to graph the resulting piecewise function.

3. What are the intervals for the original piecewise function?

4. What are the intervals for the transformed function?

THINK AND DISCUSS

5. Explain how the graph of the transformed function compares to the graph of the original function.

6. Explain how the intervals for the transformed function are related to the intervals for the original function.

EXPLORATION

9-4 Operations with Functions

Safety experts use functions to model the distance a car travels before stopping once the driver has noticed a hazard in the road. The table shows the reaction distance (the distance the car travels before the driver hits the brake pedal) and the braking distance (the distance the car travels once the brake has been applied).

Speed (mi/h)	Reaction Distance (ft)	Braking Distance (ft)
10	11	6
20	22	24
30	33	54
40	44	96
50	55	150

1. Determine a function $R(x)$ for the reaction distance, where x is the car's speed in miles per hour.
2. The function for the braking distance $B(x)$ is a quadratic function of the form $B(x) = ax^2$. Determine the value of a and write $B(x)$
3. Write a new function $T(x)$ by adding the rules for $R(x)$ and $B(x)$.

THINK AND DISCUSS

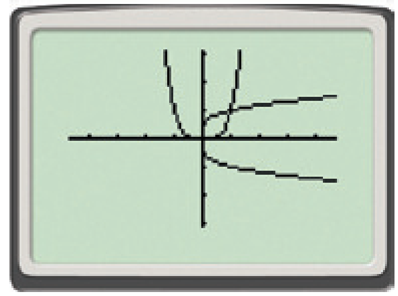
4. **Explain** what the function $T(x)$ represents and use it to find $T(60)$.
5. **Discuss** the meaning of $T(x) - B(x)$.

EXPLORATION

9-5 Functions and Their Inverses

For this Exploration, set your calculator to the decimal window by pressing ZOOM and selecting **4:ZDecimal**.

1. Enter the function $f(x) = x^4$ as **Y1** and view its graph.



2. To graph the inverse relation, return to the home screen, press 2nd DRAW PRGM, and select **8:DrawInv**. Then press VARS and use the arrows keys to move to the **Y-VARS** menu. Select **1:Function** and then **1:Y1**. Press ENTER to view the graph of $f(x)$ and its inverse.

3. Graph each function in the table. Tell whether there is a horizontal line that passes through more than one point on the graph. Then graph the inverse relation and tell whether it is a function.

Function	Is there a horizontal line that passes through more than one point on the graph?	Is the inverse relation a function?
$y = 2^x$		
$y = x^2 + 1$		
$y = x^3 + x - 1$		

THINK AND DISCUSS

4. **Discuss** any conclusions you can draw from the table.

EXPLORATION

9-6 Modeling Real-World Data

For an object dropped from a height of x feet, the function $y = \frac{\sqrt{x}}{4}$ can be used to approximate the time in seconds that will pass before the object reaches the ground.

1. The table shows the heights of various objects. Complete the table by finding the time that passes before the object reaches the ground.

Height x	16	64	144	256	400	576
Time y						

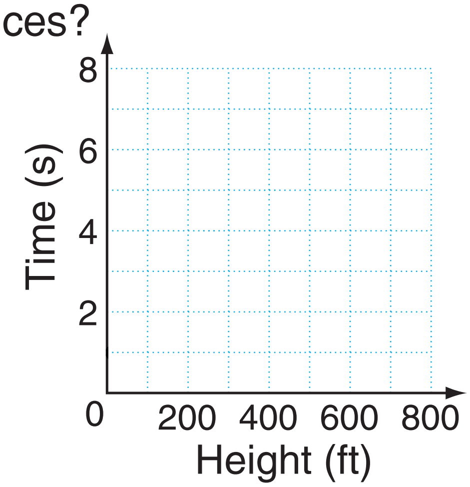
2. What do you notice about the times?

3. Find the first differences and second differences of the heights.

4. What do you notice about the differences?

5. Plot the data points on a coordinate plane.

6. Describe the general shape of the graphed data.



THINK AND DISCUSS

7. **Describe** the parent function that best models this data set.

8. **Explain** how you can recognize a square-root function from a table of values.

EXPLORATION

10-1 Introduction to Conic Sections

To graph any relation, you can begin by plotting a few points. Recall that not all graphs represent functions.

1. Complete the table for the equation $x^2 + y^2 = 25$. Be sure to find all the y-values for each x-value.

x	-5	-4	-3	0	3	4	5
y							

2. Plot the points and graph the relation.
3. What type of shape is the graph?
4. Complete the table for the equation $4x^2 + 9y^2 = 36$. Be sure to find all the y-values for each x-value. Round your answers to the nearest tenth if necessary.

x	-3	-2	-1	0	1	2	3
y							

5. Plot the points and graph the relation.

THINK AND DISCUSS

6. **Describe** the graph of $4x^2 + 9y^2 = 36$. How is it different from the graph of $x^2 + y^2 = 25$?

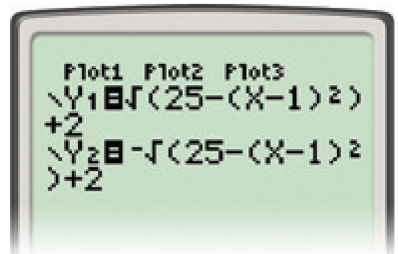
7. **Explain** whether the relations that you graphed are functions.

EXPLORATION

10-2 Circles

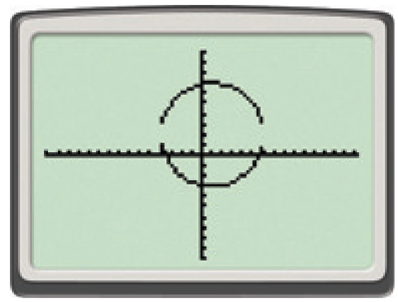
You can use your calculator to discover properties of circles.

For example, to graph $(x - 1)^2 + (y - 2)^2 = 25$, first solve for y as shown.



$$(x - 1)^2 + (y - 2)^2 = 25$$
$$(y - 2)^2 = 25 - (x - 1)^2$$
$$y - 2 = \pm\sqrt{25 - (x - 1)^2}$$
$$y = \pm\sqrt{25 - (x - 1)^2} + 2$$

Enter the positive and negative functions in your calculator as **Y1** and **Y2** and graph these in a square window. The graph shows that the center of the circle is $(1, 2)$ and the radius is 5.



1. Graph each equation of a circle. Find the center and radius of each circle to complete the table.

Equation	Center	Radius
$x^2 + (y - 3)^2 = 9$		
$(x - 5)^2 + (y + 3)^2 = 16$		
$(x + 4)^2 + y^2 = 36$		

THINK AND DISCUSS

2. **Describe** how to find the center and radius of the circle $(x + 5)^2 + (y - 9)^2 = 4$ without graphing the equation.

3. **Explain** how you know that the graph of the circle $(x + 5)^2 + (y - 9)^2 = 4$ does not intersect either axis.

EXPLORATION

10-3 Ellipses

You can discover properties of ellipses by plotting a few points and sketching the graphs.

1. Complete the table for the equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Then plot these points to help you sketch the graph of the ellipse.

x	*y*
0	
	0

2. Complete the table for the equation $\frac{y^2}{25} + \frac{x^2}{9} = 1$. Then plot these points to help you sketch the graph of the ellipse.

x	*y*
0	
	0

Use the above technique to sketch the graphs of the following ellipses. Look for patterns as you work.

3. $\frac{x^2}{16} + \frac{y^2}{4} = 1$

4. $\frac{y^2}{16} + \frac{x^2}{4} = 1$

5. $\frac{x^2}{49} + \frac{y^2}{36} = 1$

6. $\frac{y^2}{49} + \frac{x^2}{36} = 1$

THINK AND DISCUSS

7. **Describe** how the ellipses in Problems 1 and 2 are similar and how they are different.
8. **Explain** how you can look at the equation of an ellipse to decide whether the ellipse is horizontal or vertical.

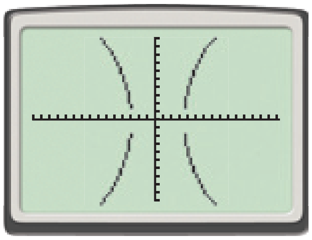
EXPLORATION

10-4 Hyperbolas

Use your calculator to investigate an equation in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

1. Solve the equation $\frac{x^2}{9} - \frac{y^2}{25} = 1$ for y.

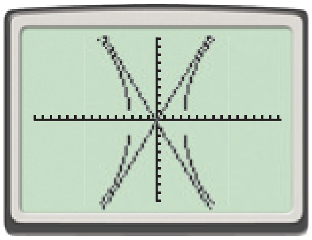
2. Graph the positive and negative parts of the equation as **Y1** and **Y2**.



3. Describe the graph.

4. What are the x-intercepts of the graph?

5. Enter $y = \frac{5}{3}x$ as **Y3** and $y = -\frac{5}{3}x$ as **Y4**. Graph these lines in the same window as the previous graph.



6. How is the graph of $\frac{x^2}{9} - \frac{y^2}{25} = 1$ related to the graph of the lines represented by $y = \pm\frac{5}{3}x$?

THINK AND DISCUSS

7. **Compare** how the graph of $\frac{x^2}{9} - \frac{y^2}{25} = 1$ to the graph of $\frac{y^2}{9} - \frac{x^2}{25} = 1$.

8. **Explain,** why the graph of $\frac{x^2}{9} - \frac{y^2}{25} = 1$ does not intersect the y-axis, without actually graphing the equation.

EXPLORATION

10-5 Parabolas

The graph of an equation in the form $y = \frac{1}{4p}x^2$ or $x = \frac{1}{4p}y^2$ is a parabola. Use your calculator to explore how the value of p affects the shape and position of the parabola.

1. Use your calculator to graph each equation in the table. (*Hint:* For equations in the form $x = \frac{1}{4p}y^2$, solve for y and graph two separate functions.) Tell whether each parabola opens upward, downward, to the left, or to the right.

Equation	Parabola Opens
$y = \frac{1}{8}x^2$	
$y = -\frac{1}{8}x^2$	
$x = \frac{1}{8}y^2$	
$x = -\frac{1}{8}y^2$	

2. Graph $y = \frac{1}{4}x^2$, $y = \frac{1}{8}x^2$, $y = \frac{1}{12}x^2$, and $y = \frac{1}{16}x^2$ in the same window. What happens to the graph as the value of p increases in the equation $y = \frac{1}{4p}x^2$?

THINK AND DISCUSS

3. **Explain** how you can tell the direction in which a parabola opens by looking at its equation.

4. **Describe** how the graph of $x = \frac{1}{32}y^2$ compares to the graph of $x = -\frac{1}{32}y^2$.

EXPLORATION

10-6 Identifying Conic Sections

The equation of any conic section can be written in the general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

1. Sketch the graph of $\frac{(x-2)^2}{9} - \frac{(y-1)^2}{25} = 1$.

2. What type of conic section is this?

3. To write the equation of the conic section in general form, first multiply both sides of the equation by 225 (the LCM of 9 and 25), as shown.

$$\frac{(x-2)^2}{9} - \frac{(y-1)^2}{25} = 1$$

$$25(x-2)^2 - 9(y-1)^2 = 225$$

Now expand the binomials and simplify to write the equation in general form.

4. What are the values of *A, B, C, D, E,* and *F* for this conic section?

THINK AND DISCUSS

5. **Explain** how to write the equation of the ellipse $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{25} = 1$ in general form.

6. **Discuss** what type of conic section is represented by an equation in general form when $A = B = 0$.

EXPLORATION

10-7 Solving Nonlinear Systems

Use a square window on your calculator for this Exploration.

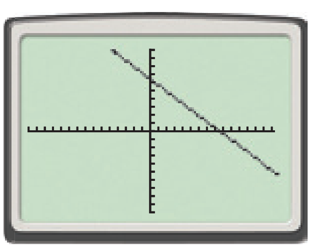
1. Graph the line $y = -\frac{3}{4}x + \frac{25}{4}$.
2. Graph the circle $x^2 + y^2 = 25$ in the same window. What can you say about the relationship between the two graphs?

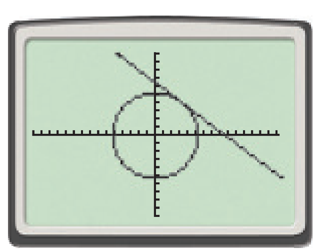
3. Use the intersect feature of your graphing calculator to find the point of intersection of the graphs. Press **2nd** **TRACE** (CALC) and select **5:intersect**. Use the arrow keys to move the cursor to the point of intersection. Note that you must use the positive part of the circle only.

4. What is the solution to the system of equations $\begin{cases} y = -\frac{3}{4}x + \frac{25}{4} \\ x^2 + y^2 = 25 \end{cases}$?
5. Use the above method to solve the system $\begin{cases} y = -\frac{3}{4}x \\ x^2 + y^2 = 25 \end{cases}$.

THINK AND DISCUSS

6. **Explain** what happens when you try to solve the system $\begin{cases} y = -\frac{3}{4}x + 7 \\ x^2 + y^2 = 25 \end{cases}$.
7. **Describe** the possible solutions for a system consisting of a line and a circle.

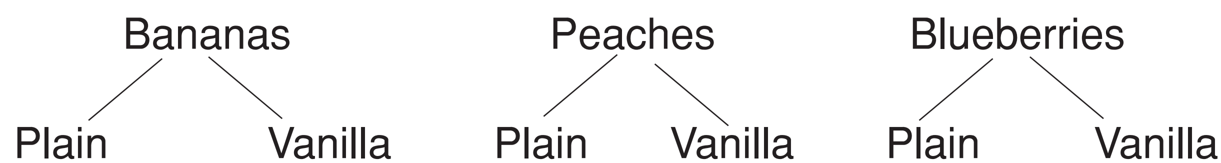




EXPLORATION

11-1 Permutations and Combinations

At a juice bar, a customer who orders a smoothie gets to choose one type of fruit and one type of yogurt. For example, customers who order Simple Smoothies get to choose from 3 fruits (bananas, peaches, or blueberries) and 2 yogurts (plain or vanilla). The tree diagram shows that there are 6 possible Simple Smoothies.



1. Use tree diagrams to help you complete the table.

Smoothie Name	Possible Fruits	Possible Yogurts	Number of Possible Smoothies
Berry Smoothie	strawberries, blueberries	raspberry, blackberry	
Choco Smoothie	bananas, raspberries, strawberries	chocolate	
Tropical Smoothie	papayas, mangos, bananas	coconut, lime, kiwi	

THINK AND DISCUSS

2. **Discuss** how the number of fruit choices and the number of yogurt choices are related to the number of possible smoothies.

3. **Explain** how you can find the number of possible smoothies when there are *m* choices for the fruit and *n* choices for the yogurt.

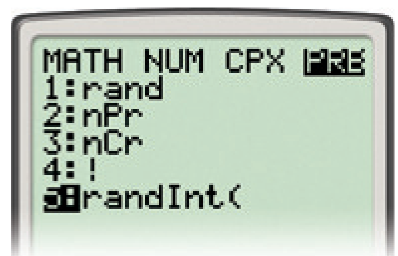
EXPLORATION

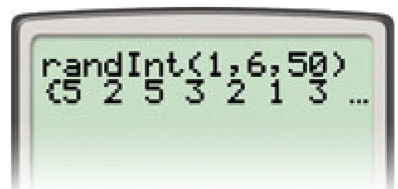
11-2 Theoretical and Experimental Probability

You can use a calculator to help you investigate probability.

1. A number cube has faces numbered 1 to 6. How many of the faces are numbered with a multiple of 3?
2. What fraction of the faces is labeled with a multiple of 3? This fraction represents the theoretical probability of rolling a multiple of 3 because it is based on *possible* outcomes.

You can model rolling a number cube by using the random integer feature of a calculator. Press MATH, scroll right to **PRB**, and select **5:randInt(.** Then enter 1 , 6 , 50) and press ENTER to see a list of 50 random integers from 1 to 6. Use the right arrow key to see all of the results. These numbers model 50 rolls of a number cube.





3. How many of the 50 rolls resulted in a multiple of 3?
4. What fraction of the 50 rolls resulted in a multiple of 3? This fraction represents the experimental probability of rolling a multiple of 3 because it is based on *actual* outcomes.

THINK AND DISCUSS

5. **Discuss** what you think will happen to the experimental probability if you increase the number of rolls.
6. **Predict** the number of rolls that will result in a multiple of 3 if a number cube is rolled 300 times. Justify your answer.

11-3 Independent and Dependent Events

Events are *independent* if the occurrence of one event does not affect the probability of the other. Events are *dependent* if the occurrence of one event does affect the probability of the other.

1. Christie has 2 dimes and 1 penny in her pocket. If she takes one coin at random, what is the probability that it will be a dime? What is the probability that it will be a penny?

Christie takes a dime from her pocket.

2. What are the possible outcomes if she takes a second coin from her pocket without replacing the first coin?

3. What is the probability that the second coin will be a dime? What is the probability that it will be a penny?

4. If Christie tosses one of her dimes, what is the probability that it will land on heads?

5. Christie's first dime lands on heads. If she tosses the second dime, what is the probability that it will land on heads? What is the probability that it will land on tails?

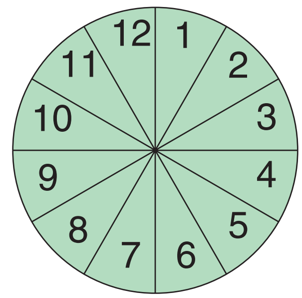
THINK AND DISCUSS

6. **Explain** why the events described in Problems 2 and 3 are dependent events.
7. **Explain** why the events described in Problems 4 and 5 are independent events.

EXPLORATION

11-4 Compound Events

The Math Club sponsors a booth at a school carnival. Visitors to the booth can win prizes by spinning the wheel shown.



Find each probability.

1. the wheel landing on an odd number
2. the wheel landing on a multiple of 4
3. the wheel landing on a prime number
4. During the morning, a player can win a prize if the player's spin lands on an odd number or a multiple of 4. Which numbers result in a player winning a prize?
5. What is the probability that a player will win a prize in one spin?
6. During the afternoon, a player can win a prize if the player's spin lands on an odd number or a prime number. Which numbers result in a player winning a prize?
7. What is the probability that a player will win a prize in one spin?

THINK AND DISCUSS

8. **Explain** whether the two possible winning events in the morning game are mutually exclusive. *Mutually exclusive events* cannot both occur in the same trial of an experiment.
9. **Explain** whether the two possible winning events in the afternoon game are mutually exclusive.

EXPLORATION

11-5 Measures of Central Tendency and Dispersion

The advertised mass of a box of cereal is 453 grams. A quality control manager measures 13 boxes and records the following masses in grams.

450, 453, 451, 456, 455, 454, 456, 452, 452, 453, 495, 453, 456

Enter the data into a list in a calculator by pressing STAT and selecting **1:Edit...**. Then enter the data into list **L1**. Press 2nd MODE (QUIT) to return to the home screen.

1. Find the average, or mean, of the data by pressing 2nd STAT (LIST), scrolling right to **MATH**, and selecting **3:mean(**. Then press 2nd 1 (L1)) ENTER .
2. Find the middle value, or median, of the data by using the procedure described above but select **4:median(** instead of **3:mean(**.
3. An outlier is a value that is much less than or much greater than the other values in a data set. Which value in the data set above is most likely to be considered an outlier?
4. How does the mean of the data set change if this outlier is removed? How does the median change?

THINK AND DISCUSS

5. **Discuss** whether the mean or median gives a better measure of the center of the data set.

EXPLORATION

11-6 Binomial Distributions

The figure shows the first few rows of Pascal's triangle. Each row begins and ends with 1. Any other entry in a row is the sum of the two entries to its upper left and upper right.

Row 0	1
Row 1	1 1
Row 2	1 2 1
Row 3	1 3 3 1

1. Write row 4 and row 5 of Pascal's triangle.
2. Recall that ${}_nC_r = \dfrac{n!}{r!(n-r)!}$. Calculate ${}_3C_0$, ${}_3C_1$, ${}_3C_2$, and ${}_3C_3$. How do these combinations relate to Pascal's triangle?
3. Calculate ${}_4C_0$, ${}_4C_1$, ${}_4C_2$, ${}_4C_3$ and ${}_4C_4$. How do these combinations relate to Pascal's triangle?
4. List the combinations that could be used to determine row 5 of Pascal's triangle.

THINK AND DISCUSS

5. **Discuss** whether the pattern of combinations holds for row 0 and row 1 of Pascal's triangle.
6. **Explain** how you could determine row n of Pascal's triangle, where n is a whole number.

EXPLORATION

12-1 Introduction to Sequences

A graphic designer uses software to enlarge a square repeatedly. The original dimensions of the square (stage 1) are 10 inches by 10 inches. The table shows the square's area as it is enlarged.

Stage	1	2	3	4	5
Area (in^2)	100	110	121	133.1	146.41

1. Look for a pattern in the table. How is each value in the list of areas related to the previous one?
2. By how much does the software enlarge the square at each stage?
3. Find the area of the square at the next three stages.
4. What type of function models the data in the table? Why?
5. What is the domain of the function in this situation?

THINK AND DISCUSS

6. **Explain** how you can find the area of the square at stage 10.
7. **Explain** how you can find the area of the square at stage 11 once you know the area of the square at stage 10.

EXPLORATION

12-2 Series and Summation Notation

The sequence given by the explicit formula $a_n = 2n - 1$ has some interesting properties.

1. Complete the table to find the first 5 terms of the sequence $a_n = 2n - 1$.

n	1	2	3	4	5
a_n					

2. Describe the sequence.

3. This table shows the sum of the first n terms of the above sequence. Complete the table.

n	1	2	3	4	5
Sum of first n terms of the sequence $a_n = 2n - 1$	1	4			

4. Describe any patterns you see in the table.

5. Predict the sum of the first 6 terms of the sequence $a_n = 2n - 1$.

THINK AND DISCUSS

6. **Explain** how you can use what you discovered to find the sum of the first 15 odd numbers.

7. **Describe** a sequence for which the sum of its terms is 81.

12-3 Arithmetic Sequences and Series

Bryan joins a DVD club that sends him 8 DVDs the first month and 3 DVDs every month thereafter.

1. Complete the table showing the number of DVDs Bryan buys through the club.

Month	Total Number of DVDs
1	8
2	
3	
4	
5	

2. Graph the data in the table.
3. What is the difference of the successive terms in the sequence formed by the total number of DVDs?
4. What type of function describes the data? Why?
5. Write an explicit formula for the *n*th term of the sequence formed by the total number of DVDs.

THINK AND DISCUSS

6. **Show** how you can use your rule to find the number of DVDs Bryan will have bought after 12 months.
7. **Discuss** how you can use your rule to determine whether or not Bryan will ever have bought exactly 75 DVDs.

EXPLORATION

12-4 Geometric Sequences and Series

This year, Kate buys a used car worth $10,000. In each subsequent year, the car's value is 90% of the previous year's value.

1. Complete the table showing the yearly values of Kate's car.

Year	Value of Car ($)
1	10,000
2	
3	
4	
5	

2. Graph the data in the table.
3. What is the ratio of the successive terms in the sequence formed by the yearly values of Kate's car?
4. What type of function describes the data? Why?
5. Write an explicit formula for the *n*th term of the sequence formed by the yearly values of Kate's car.
6. Write a recursive formula for the *n*th term of the sequence.

THINK AND DISCUSS

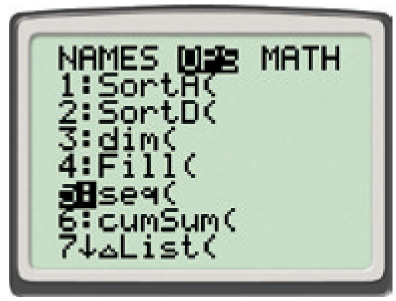
7. **Show** how you can use the explicit formula to find the value of Kate's car in the 8th year.
8. **Discuss** whether the explicit formula or the recursive formula is more useful.

EXPLORATION

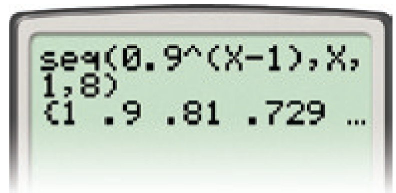
12-5 Mathematical Induction and Infinite Geometric Series

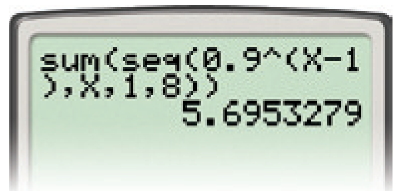
Use your calculator to explore the sequence $a_n = 0.9^{n-1}$.

1. You can display the first 8 terms of the sequence as follows. Press 2nd LIST STAT and scroll to the **OPS** menu. Select **5:seq(**. Then enter the rule for the sequence as shown. Close the parentheses and press ENTER. Use the arrow keys to scroll right to see all the terms.



2. You can find the sum of the first 8 terms as follows. Press 2nd LIST STAT and scroll to the **MATH** menu. Select **5:sum(**. Then enter the sequence as in Step 1, close the parentheses, and press ENTER.





3. Use your calculator to find the sum of the first 20 terms of the sequence. Then find the sum of the first 50 terms, the first 100 terms, and the first 200 terms.

THINK AND DISCUSS

4. Describe what happens to the partial sums of the sequence as you add more and more terms.

5. Discuss what you think would happen if it were possible to add all of the infinitely many terms of the sequence.

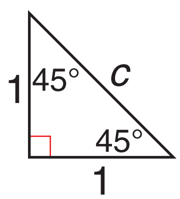
EXPLORATION

13-1 Right-Angle Trigonometry

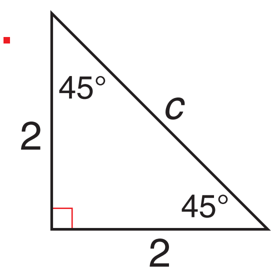
You can use the Pythagorean Theorem to help you discover properties of special right triangles.

Use the Pythagorean Theorem to find the length of the hypotenuse *c* of each right triangle. Write your answers in simplest radical form.

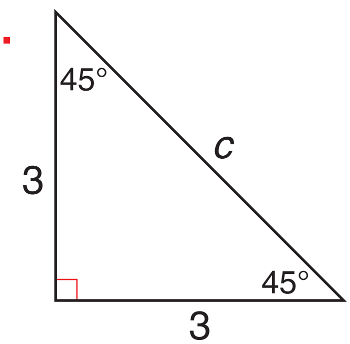
1.



2.



3.



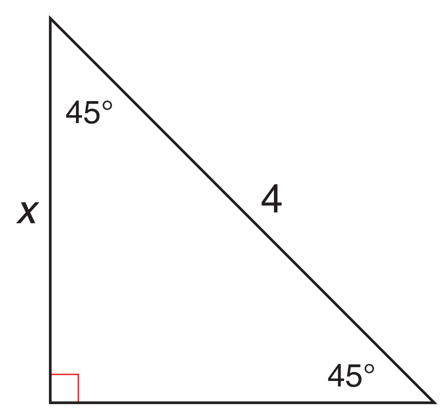
4. For each triangle in Problems 1–3, find the ratio of the length of a leg to the length of the hypotenuse.

5. What pattern do you notice about the ratios you calculated?

THINK AND DISCUSS

6. Describe how the side lengths are related to each other in a 45°-45°-90° triangle.

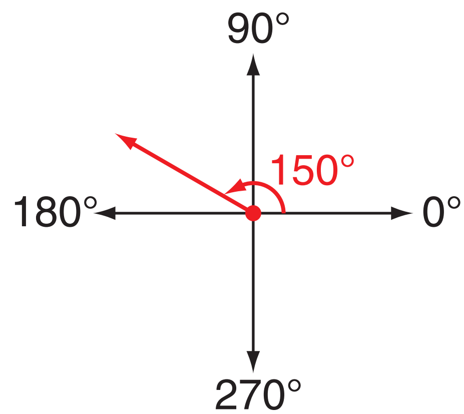
7. Explain how you can use the ratio you discovered above to find the value of x in this triangle.



EXPLORATION

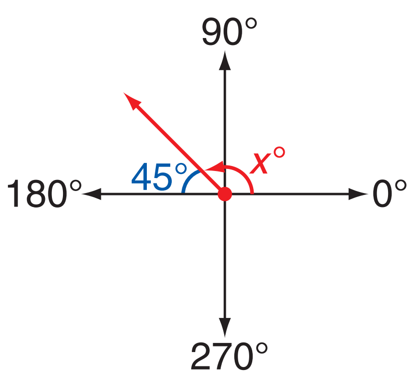
13-2 Angles of Rotation

Imagine a ray on the positive *x*-axis with its vertex at the origin. As the ray rotates counterclockwise about the origin, you can use what you know about angle measures to determine the number of degrees through which the ray rotates.



Find the number of degrees *x* through which each ray has rotated. (*Hint*: A complete rotation is equal to 360°.)

1.



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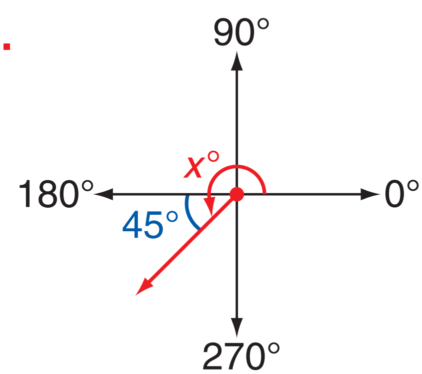

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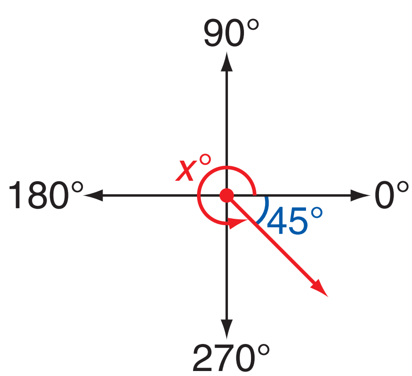

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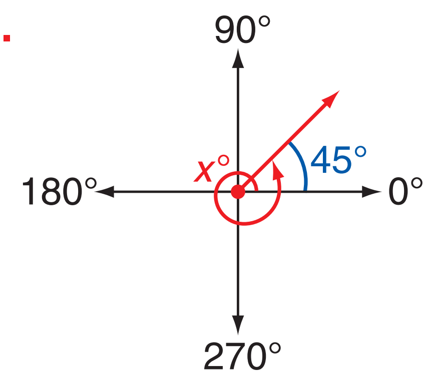


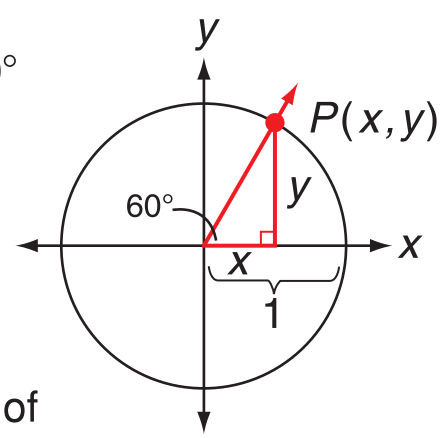
THINK AND DISCUSS

5. **Explain** how you found the number of degrees through which the ray rotated in Problems 1–4.

EXPLORATION

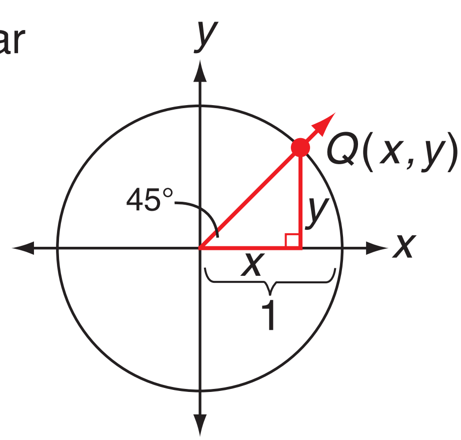
13-3 The Unit Circle

A *unit circle* is a circle with a radius of 1 unit. You can use trigonometric ratios to determine the coordinates of points on a unit circle that is centered at the origin.



1. The figure shows a unit circle and a 60° angle in standard position. What is the length of the hypotenuse of the 30°-60°-90° triangle? How do you know?
2. Use what you know about special right triangles to find the lengths of the legs of the 30°-60°-90° triangle.
3. What are the coordinates of point *P*?
4. What are the exact values of cos 60° and sin 60°?
5. How are the values of cos 60° and sin 60° related to the coordinates of point *P*?

THINK AND DISCUSS



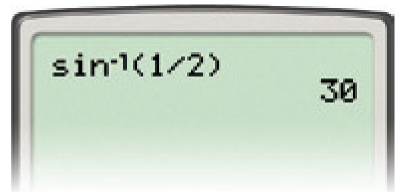
6. **Describe** how you can use a similar method to find the coordinates of point *Q*.
7. **Explain** how the values of cos 45° and sin 45° are related to the coordinates of point *Q*.

EXPLORATION

13-4 Inverses of Trigonometric Functions

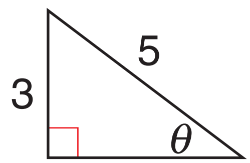
The inverse sine feature of a graphing calculator can be used to determine the measures of acute angles in right triangles.

To find the measure of an angle that has a sine of $\frac{1}{2}$, press 2nd SIN (SIN^{-1}) and enter $\frac{1}{2}$ as shown. The calculator shows that a 30° angle has a sine of $\frac{1}{2}$.



For each triangle, write the exact value of the sine ratio for θ. Then use the inverse sine feature to find the value of θ to the nearest tenth of a degree.

1.



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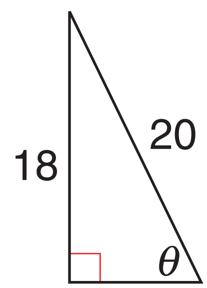
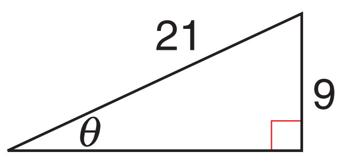
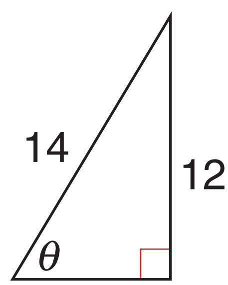


3.

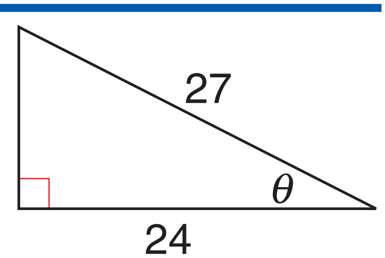


4.



THINK AND DISCUSS

5. **Discuss** how you could use the inverse cosine feature of a calculator to find the value of θ in the triangle shown.



EXPLORATION

13-5 The Law of Sines

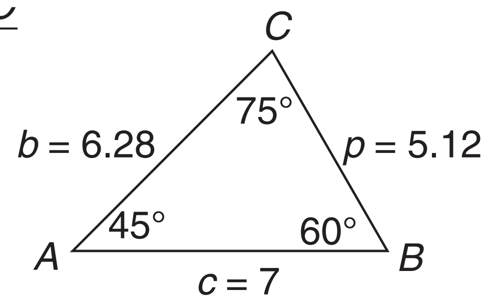
You can use a calculator to investigate the relationship between the sines of the angles of a triangle and the lengths of the opposite sides.

Use your calculator to find the value of each ratio. Round to the nearest thousandth.

1. $\frac{\sin A}{a}$

2. $\frac{\sin B}{b}$

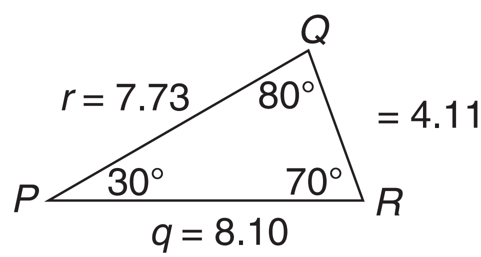
3. $\frac{\sin C}{c}$



4. $\frac{\sin P}{p}$

5. $\frac{\sin Q}{q}$

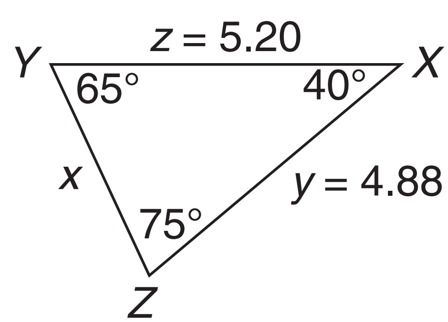
6. $\frac{\sin R}{r}$



THINK AND DISCUSS

7. **Describe** the pattern you notice in the ratios for each triangle.

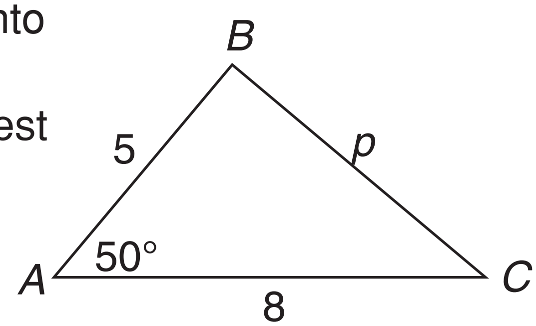
8. **Discuss** how you can use the pattern you observed to find the value of *x*.



EXPLORATION

13-6 The Law of Cosines

The lengths of two sides and the measure of the included angle are given for $\triangle ABC$. To solve this triangle, you must use the Law of Cosines. For $\triangle ABC$, the Law of Cosines states that $a^2 = b^2 + c^2 - 2bc \cos A$.



1. Substitute the known measures into the Law of Cosines and solve for the value of *a*. Round to the nearest tenth.

2. Use the Law of Sines to find the measure of $\angle C$ to the nearest tenth of a degree.

3. Find the measure of $\angle B$ to the nearest tenth of a degree. Explain how you determined your answer.

THINK AND DISCUSS

4. **Explain** why you cannot use the Law of Sines to find the value of *a* in $\triangle ABC$.

5. **Describe** how you could use the Law of Sines to check that you solved $\triangle ABC$ correctly.

EXPLORATION

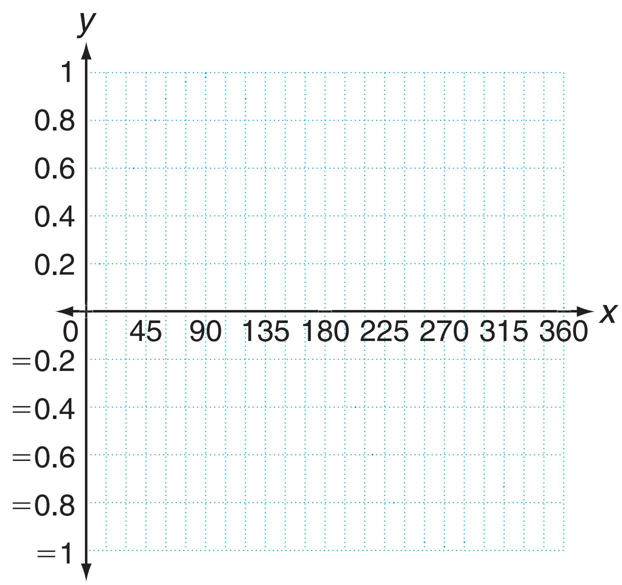
14-1 Graphs of Sine and Cosine

You can investigate the graph of the function $y = \sin x$ by making a table of values and plotting points.

1. Use your calculator to complete the table for the function $y = \sin x$. Round each y-value to the nearest hundredth if necessary.

$x(°)$	0, 360	30	45	60	90	120	135	150
y								
$x(°)$	180	210	225	240	270	300	315	330
y								

2. Plot the points from your table on a coordinate plane like the one shown. Connect the points to form a smooth curve.



THINK AND DISCUSS

3. Describe the maximum and minimum values of the function.

4. Discuss what will happen if you continue the graph another 360° to the right.

EXPLORATION

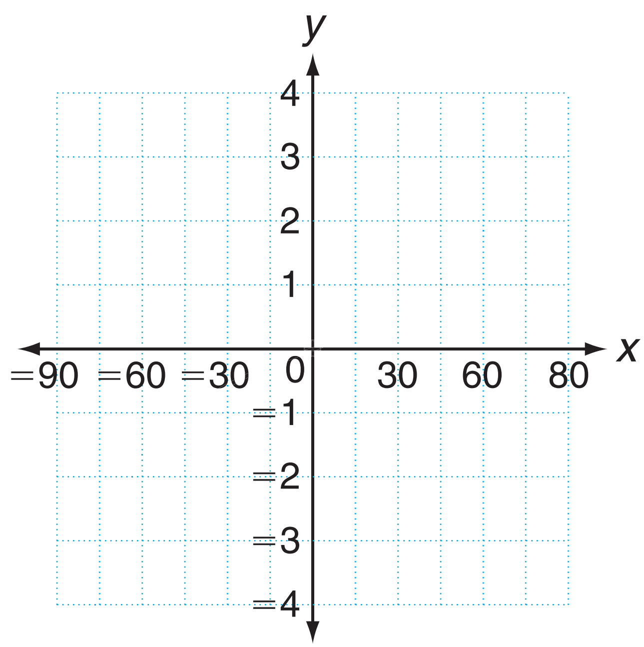
14-2 Graphs of Other Trigonometric Functions

You can investigate the graph of the function $y = \tan x$ by making a table of values and plotting points.

1. Use your calculator to complete the table for the function $y = \tan x$. Round each y-value to the nearest hundredth if necessary.

$x(°)$	-90	-60	-45	-30	0	30	45	60	90
y									

2. Plot the points from your table on a coordinate plane like the one shown. Connect the points to form a smooth curve.



THINK AND DISCUSS

3. **Explain** what happens when x approaches 90° or $-90°$.

4. **Describe** what you think would happen if you continued the graph another 180° to the right.

EXPLORATION

14-3 Fundamental Trigonometric Identities

You can discover properties of trigonometric functions by evaluating expressions.

Evaluate each of the following expressions.

1. $\sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{6}\right)$

2. $\sin^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{4}\right)$

3. $\sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right)$

4. What do you notice about the expressions in 1–3?

5. Enter the function $y = \sin^2 x + \cos^2 x$ into your calculator and view a table of values. What do you find?

THINK AND DISCUSS

6. **Derive** a new identity by investigating the expression $\sec^2 x - \tan^2 x$.

7. **Derive** a new identity by investigating the expression $\cot^2 x - \csc^2 x$.

EXPLORATION

14-4 Sum and Difference Identities

You can use patterns to discover new trigonometric identities.

1. In each row of the table, the measures of two angles, $\angle A$ and $\angle B$, are given. Use your calculator to evaluate $\sin(A + B)$, $\sin A \cos B$, and $\cos A \sin B$. Round each value to the nearest thousandth if necessary.

$m\angle A$	$m\angle B$	$\sin(A + B)$	$\sin A \cos B$	$\cos A \sin B$
10°	20°			
25°	40°			
50°	15°			
60°	20°			
100°	150°			

2. Look for a pattern in the table. How is $\sin(A + B)$ related to $\sin A \cos B$, and $\cos A \sin B$?

3. Write an identity based on your answer to Problem 2.

THINK AND DISCUSS

4. **Explain** what happens when you use your identity to evaluate $\sin(45° + 45°)$.

5. **Show** how you can derive an identity for $\sin(A - B)$ by replacing B with $-B$ in the identity you discovered and using the fact that $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$.

EXPLORATION

14-5 Double-Angle and Half-Angle Identities

It is often useful to rewrite trigonometric expressions involving 2θ in terms of θ only. You can use your calculator to develop an identity for $\sin 2\theta$.

1. In each row of the table, the measure of angle θ is given. Use your calculator to find $\sin 2\theta$ and $\sin\theta\cos\theta$. Round each value to the nearest thousandth if necessary.

θ	$\sin 2\theta$	$\sin\theta\cos\theta$
10°		
30°		
65°		
80°		
150°		

2. Look for a pattern in the table. How is $\sin 2\theta$ related to $\sin\theta\cos\theta$?

3. Write an identity based on your findings.

THINK AND DISCUSS

4. **Explain** how you could use a graph or table to verify the identity you discovered.

5. **Describe** what happens when $\theta = 45°$ in the identity.

EXPLORATION

14-6 Solving Trigonometric Equations

You can use your calculator to help you solve trigonometric equations. Set your calculator to the trigonometry window by pressing ZOOM and selecting **7:ZTrig**. The *x*-axis will now display values from -2π to 2π if you are in **RADIAN** mode.

1. Solve the equation $\cos\theta = -1$ for values of θ within this viewing window by entering the functions $y = \cos\theta$ and $y = -1$ and finding all the points of intersection.

2. How many solutions are there if you do not restrict the possible values of θ? What are the solutions?

3. Use the same method to find *all* the solutions of the equation $\sin\theta\cos\theta = -\sin\theta$.

4. You can also solve $\sin\theta\cos\theta = -\sin\theta$ algebraically by adding $\sin\theta$ to both sides of the equation and then factoring. Show that this method results in the same set of solutions.

THINK AND DISCUSS

5. **Explain** how the solutions of $\cos\theta = -1$ are related to the period of the cosine function.

6. **Explain** why you cannot solve $\sin\theta\cos\theta = -\sin\theta$ by simply dividing both sides by $\sin\theta$ and then solving $\cos\theta = -1$.